

A Method of Weighted Mean for Structural Random Loading Identification

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Abstract

Multi-input-multi-output stationary random loading identification has been long a difficult problem for its poor identification precision caused by ill-condition problem of transfer function matrix inversion. In this paper, the inverse pseudo-excitation method is used for identification problems. A new condition number weighted average method is developed to improve the accuracy of loading identification. A threshold value is introduced to reduce the computation cost. Simulations and experiments are designed to certify the feasibility of the algorithm and achieve good results. The condition number of FRF is introduced to choose the response measuring point in the experiment to reduce the workload and improves the identification precision.

Keywords: condition number, loading identification, inverse pseudo-excitation method, identification experiment, random vibration, method of weighted mean.

1. Introduction

Random vibrations exist in process of ground transportation, high-speed flight phase of the aircraft, boost phase of rocket, and so on. It receives increasing attention. For multiple stationary random excitations, the direct problem is the computation of the power spectral densities (PSDs) of the various responses from the given excitation PSD. It has been solved (Robson, 1964; Piersol, 1993) efficiently by many classic method, and pseudo excitation method (PEM) proposed by Lin et al (Lin, J. H. 1992; Lin, J. H. 1994) is a efficient one. There are two types of inverse problems (also called back analysis problems) corresponding to this direct problem. The first is the so called structure identification problem, for which the loadings and response PSD functions are all known and are used to identify the properties of the structure. System identification and modal testing have received much attention and many publications deal with such problems and their successful applications in engineering (Juang, 1994; Ewins, 1986). The second is the so-called loading identification problem, for which the response information and structural properties are known and are used to identify information about the loading.

However, loading identification has received considerably less attention in the technical literature; even some similar works have been done at a high expense, but have rarely yielded satisfactory identification results because of their poor precision, especially for random excitations. In fact, this is a very important field, and many loads in engineering applications need to be identified. For instance, the traffic loads of bridges, the seismic excitations of buildings, the interaction forces between moving machines and their bases, and so on. It is very difficult, or sometimes impossible, to measure such excitations directly, whereas the measurement of some of the responses are relatively easy.

In this paper, the locations of the exciting points are known, and if there is only one excitation acting on the structure, the problem is quite straightforward and is not discussed. For multi-input-multi-output (MIMO) problems, however, its solution is not easy and it is difficult to find papers in this area which demonstrate its difficulty. To the authors' knowledge, some such work has been done at high expense, but has yielded identification results that are unacceptable because of their

very poor precision. The inverse pseudo-excitation method (IPEM)(Lin, J. H.,2001) ,which is a counterpart or an inversion form of PEM, is an efficient approach for the loading identification of a structure subjected to stationary random excitations. IPEM is used in this paper and condition number weighted average method is presented to improve the rank defect of the frequency response function for random loading identification. Simulation process is given to compare the presented method with the common frequency domain approach, the direct inversion method. Experiments are designed to certify the feasibility of the method, and achieved good results. In paper (Papadopoulos, 1998) a method is introduced to select sensors based on the modal kinetic energy (MKE) distribution, which gives a measure of the dynamic contribution of each finite element modal (FEM) physical degree of freedom to each of the target mode shapes. This method is also applied in the paper(Li, D. S.,(2003), and the method is used in the simulation before the experiment, and helps to set sensor in the experiment. However, errors exist in the finite element model and original systems. In this paper the condition number of FRF is used to choose the response measuring point, it reduces the computation workload, and gets a relative accurate result.

2. Method of loading identification

In this paper, loading identification is based on some assumptions as follows. The system is linear, the load positions are known and not moving, the measured responses of the structure are caused entirely by the random load to be identified, and does not exist other unknown type of excitation form acting on the system.

2.1 Direct inversion method (DIM)

The motion equations of structure can be expressed as

$$M\ddot{y} + C\dot{y} + Ky = F \quad (1)$$

In which, M , C and K are, respectively, the mass, damping and stiffness matrices, dots denote differentiation with respect to time and y and F are, respectively, the displacements and applied force vectors. The order of the system is n . m stand for the number of measured responses in y . l stand for the number of non-zero excitations in F . m and l are both less than or equal to n . $l \leq m$ is the necessary condition for the solvability of the inverse problem. According to the linear system random response power spectrum formula(Paez, T. L.,2008):

$$S_{yy} = H^* S_{FF} H^T \quad (2)$$

In which, S_{yy} is the PSD matrix of measured response, S_{FF} is the PSD matrix of the excitation, and H is the frequency response function matrix. H is in general not a square matrix, and the number of rows is greater than or equal to the number of columns. The superscripts "*" and "T" represent complex conjugate and transpose, respectively. S_{FF} is the matrix to be identified, and it can be obtained by a matrix transformation:

$$S_{FF} = H^{+*} S_{yy} H^{+T} \quad (3)$$

In which, the superscript + represents generalized inversion. The generalized inverse matrix H^+ can be obtained by direct inversion method (Horn,1990):

$$H^+ = (H^T H^*)^{-1} H^T \quad (4)$$

2.2 Condition number weighted average method (CNWAM)

The response PSD matrix S_{yy} can be decomposed as follows(Lin, J. H.,2001;Li, D. S.,2001; Li, D. S.,2003)

$$S_{yy} = \sum_{s=1}^r b_s^* \cdot b_s^T, (s=1, 2...r) \quad (1)$$

$$\mathbf{b}_s = \sqrt{\lambda_s} \cdot \boldsymbol{\psi}_s^* , (s=1, 2\dots r) \quad (2)$$

In which λ_s , $\boldsymbol{\psi}_s$ and r represent eigenvalue, eigenvector and rank number of matrix \mathbf{S}_{yy} , respectively. The pseudo-response was defined as $\tilde{\mathbf{y}}_s = \mathbf{b}_s e^{it}$, where i is the imaginary unit. Then the pseudo-excitation $\tilde{\mathbf{x}}_s$ can be got by the equation as bellow.

$$\tilde{\mathbf{x}}_s = \mathbf{H}^+ \tilde{\mathbf{y}}_s = \mathbf{H}^+ \mathbf{b}_s e^{it} (s=1, 2\dots r) \quad (3)$$

Hence Eq. (5) and (6) enable the excitation PSD matrix \mathbf{S}_{xx} to be given in terms of the $\tilde{\mathbf{x}}_s$ as

$$\mathbf{S}_{xx} = \sum_{s=1}^r \tilde{\mathbf{x}}_s^* \cdot \tilde{\mathbf{x}}_s^T = \mathbf{H}^{+*} \cdot \left(\sum_{s=1}^r \mathbf{b}_s^* \cdot \mathbf{b}_s^T \right) \cdot \mathbf{H}^{+T} \quad (4)$$

For large DOFs systems, direct using of Eq. (8) is still computationally very expensive. Therefore, mode-superposition method can be used to reduce the workload. The main error is caused by calculation of generalized inversion \mathbf{H}^+ . The condition number weighted average method (CNWAM) is developed here trying to reduce the impact of ill-conditioning problems of \mathbf{H}^+ . Eq. (7) can be written as:

$$\tilde{\mathbf{x}}_s = \mathbf{H}^+ \cdot \tilde{\mathbf{y}}_s = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \\ \dots \\ \mathbf{R}_m \end{bmatrix}^+ \cdot \begin{bmatrix} \tilde{\mathbf{y}}_1 \\ \tilde{\mathbf{y}}_2 \\ \tilde{\mathbf{y}}_3 \\ \dots \\ \tilde{\mathbf{y}}_m \end{bmatrix}_s \quad (5)$$

Where \mathbf{R}_i ($i=1,2,\dots,m$) is the row vector of $\mathbf{H}^{m \times l}$. A square matrix $\mathbf{H}_j^{l \times l}$ ($j=1, 2, 3\dots k$) can be made up by choosing number l of row vectors \mathbf{R}_i form \mathbf{H} , and this combination number is $k = C_n^m$. Accordingly, number l of $\tilde{\mathbf{y}}_i$ in the same rows of $\tilde{\mathbf{y}}_s$ are selected to assemble $\tilde{\mathbf{y}}_s^j$ ($j=1, 2\dots k, s=1, 2\dots r$). Since \mathbf{H}_j is a $l \times l$ square matrix, and generally its inverse matrix \mathbf{H}_j^{-1} exists and easy to obtain, then, $\tilde{\mathbf{x}}_s^j$ can be obtained by:

$$\tilde{\mathbf{x}}_s^j = \mathbf{H}_j^{-1} \cdot \tilde{\mathbf{y}}_s^j , (j=1, 2\dots k, s=1, 2\dots r) \quad (6)$$

q_j was defined as:

$$q_j = \frac{1}{\text{cond}(\mathbf{H}_j)} , (j=1, 2\dots k) \quad (7)$$

Where, "cond" is the operation of calculating condition number of matrix:

$$\text{cond}(\mathbf{A}) = \|\mathbf{A}\|_p \cdot \|\mathbf{A}^{-1}\|_p \quad (8)$$

In which, $\|\cdot\|_p$ is the operation of matrix norm, especially, in this paper, 2–norm is used to obtain $\text{cond}(\mathbf{A})$

The weights can be defined as follows

$$w_j = \begin{cases} q_j, q_j \geq q \\ 0, q_j < q \end{cases} (j=1, 2\dots k) \quad (9)$$

Where, q is threshold value:

$$q = \frac{1}{k} \sum_{j=1}^k q_j , (j=1, 2\dots k) \quad (10)$$

The pseudo-excitation $\tilde{\mathbf{x}}_s$ can be obtained as follows:

$$\tilde{\mathbf{x}}_s = \frac{\sum_{j=1}^k w_j \cdot \tilde{\mathbf{x}}_s^j}{\sum_{j=1}^k w_j} (s=1, 2\dots r) \quad (11)$$

Then, S_{xx} can be obtained by Eq. (8). The weighted average method tries to reduce the impact of the sub matrices with big condition numbers. The threshold q is used for judging which sub matrices should be selected to compute the pseudo excitation, and which should be discarded.

3. Computer simulation example

A 9 DOFs system is given as follows in Fig. 1. k_i ($i=1,2,\dots,9$) are stiffness coefficients of each floor, and their values are shown in Fig. 1. The first 9 order natural frequencies of the structure are shown in Table 1, and the damping ratio is set as 0.02.

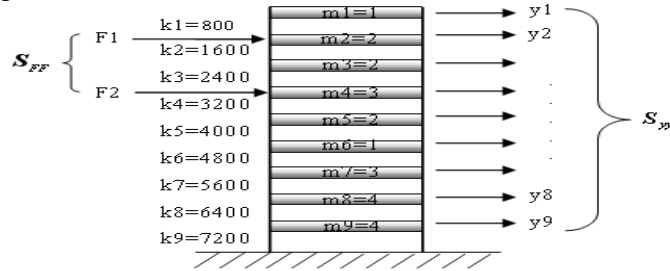


Fig. 1. 9 DOFs shear type model

The stationary random excitations are applied to m_2 and m_4 , where ρ is the correlation coefficient of the 2 forces F_1 and F_2 as shown in Fig.1, their auto-power spectral density are $S_1(\omega)$ and $S_2(\omega)$ as shown in Fig.2 and Fig.4 with solid line, respectively. S_{12} and S_{21} are the cross-spectral density of the 2 forces, which are defined by Eq. (17). The excitation PSD matrix then can be given as follows:

$$S(\omega) = \begin{bmatrix} S_1(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_2(\omega) \end{bmatrix} \quad (1)$$

$$S_{12} = \sqrt{S_1(\omega)S_2(\omega)}\rho e^{i\omega\Delta t}, \quad S_{21} = \sqrt{S_1(\omega)S_2(\omega)}\rho e^{-i\omega\Delta t} \quad (2)$$

In which, $\rho = 0.7$, $\Delta t = 0.08$.

Table 1. First 9 order natural frequencies of the structure (rad/s)

Order	1	2	3	4	5	6	7	8	9
Natural Frequency (rad/s)	19.96	39.85	42.56	45.365	58.70	59.96	93.96	116.71	119.89

Response nodes combination (3, 5, 7) is used to identify the excitation PSD at (2, 4). The response PSD matrix S_{yy} can be obtained easily, and it is used to identify the excitation PSD matrix in equation (16) using the direct inversion method (DIM) and the represented method (CNWAM) in this paper. The simulation results are represented in Fig. 2 ~ 9, and the relative errors of the two algorithms are shown in Table 2.

Table 2. Relative errors of two algorithms

method	Errors by CNWAM	Errors by DIM
Identification error of S_1	0.408%	12.02%
Identification error of S_2	0.384%	11.94%
Identification error of real part of S_{12}	0.284%	6.65%
Identification error of imaginary part of S_{12}	0.338%	9.93%

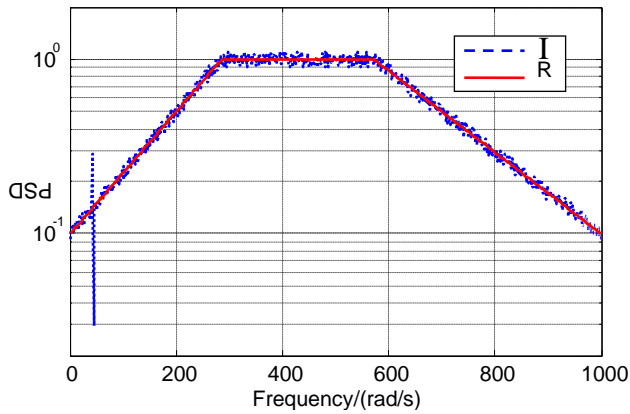


Fig. 2. Identification result of S1 by DIM

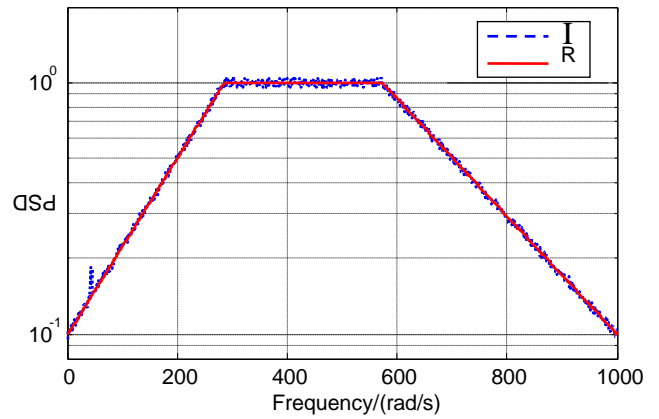


Fig. 3. Identification result of S1 by CNWAM

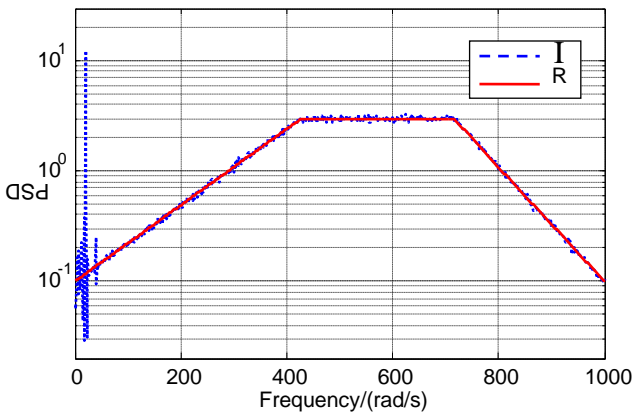


Fig. 4 Identification result of S2 by DIM

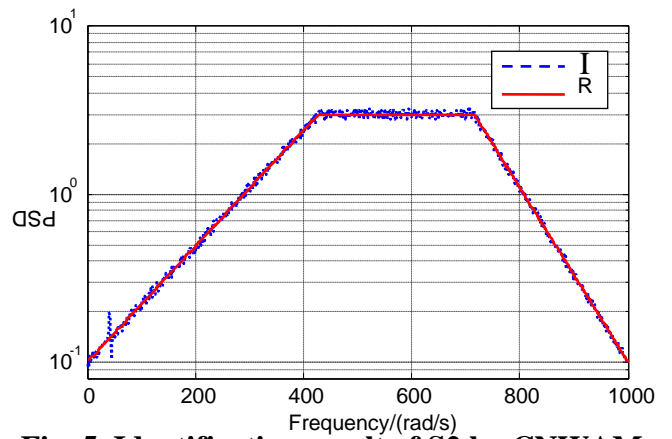


Fig. 5. Identification result of S2 by CNWAM

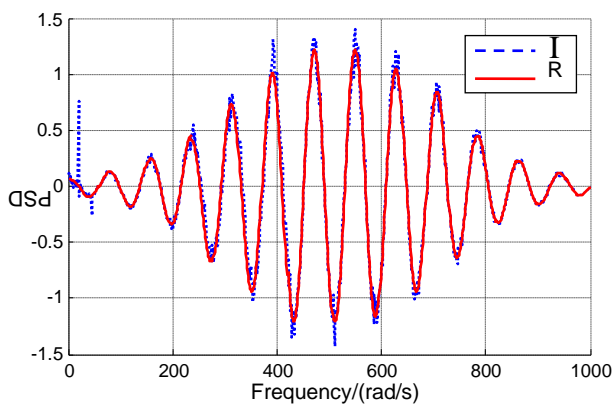


Fig. 6. Identification result of real part of S12 by DIM

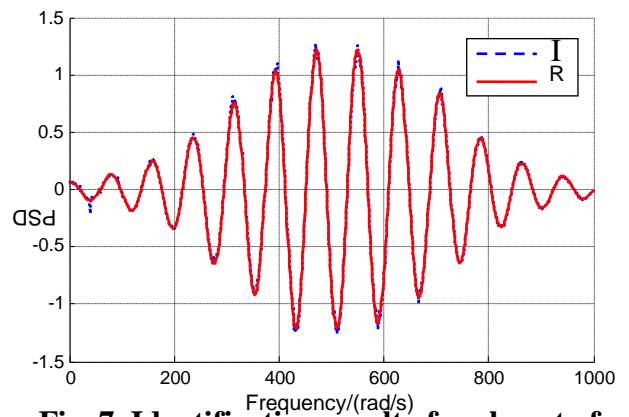


Fig. 7. Identification result of real part of S12 by CNWAM

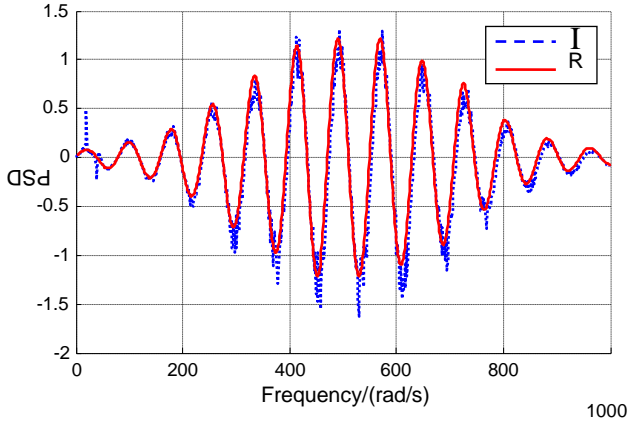


Fig. 8. Identification result of imaginary part of S12 by DIM

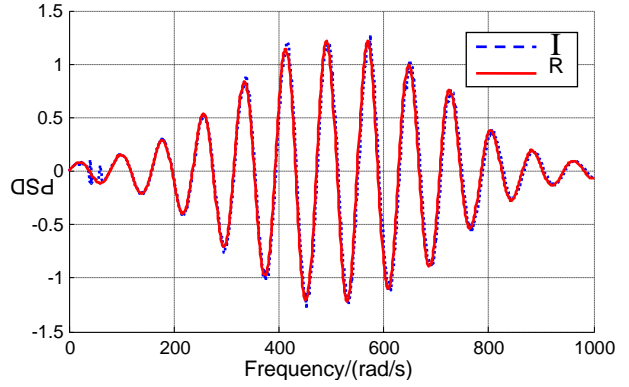


Fig. 9. Identification result of imaginary part of S12 by CNWAM

In Fig. 2~9, "I" represents result of identification value, and "R" represents theoretical value.

The method of DIM is subject to greater simulation errors than the proposed method as shown in Fig. 2-9 and table 2. Especially, in frequency ranges close-by low nature, the greater calculation errors arise by employing DIM because of inversion of ill-conditioned matrix. The simulation demonstrates the proposed approach is effective to reduce the errors and improve the identification precision. However, Algorithm cannot eliminate the environmental noise and measurement errors, especially, if the measuring point selection is not appropriate. Besides, more matrix inversion need to be done while using CNWAM, it increases the computation load.

4. Identification experiment

Experiment is conducted on the aluminum sheet shown in Fig. 10. The scale of acceleration transducer is shown in Table 3. Two random excitations and two force sensors are applied at T1 and T2. 6 acceleration sensors are applied at A3~A8. Some of the 6 acceleration response measure points are chosen to identify the random excitation and the rule of selecting response points is based on condition number of $H_{m \times l}$, and $H_{m \times l}$ is obtained by the follow equation :

$$H = G_{YF} G_{FF}^{-1} = YF^* (FF^*)^{-1} \quad (1)$$

In which, Y is the frequency domain vector obtained by applying Fast Fourier Transform Algorithm(FFT) to their time-domain signals of displacement response, F is the frequency domain vector of the excitation force, and their time-domain signals are obtained by the experiment measurement.

The real force excitations are obtained by force sensors and used to assess the proposed approach. The response combinations, (A5, A7, A8) and (A6, A7, A8), are respectively used to identify the excitation force PSD matrix. The condition number is defined as Eq.(12), and the curves of condition number of frequency response function (FRF) $H_{m \times l}(f)$ for the 2 combinations are shown in Fig. 11.

Table 3. Scale of acceleration transducer

Type	Sensitivity	Frequency Range $\pm 10\%$	Measuring range	Temperature range	Weight
333B30	100mv/g	0.5Hz~3KHz	$\pm 50g$ pk	-18~+66°C	4g
353B32	50mv/g	0.7~8000Hz	$\pm 100g$ pk	-54~121°C	20g

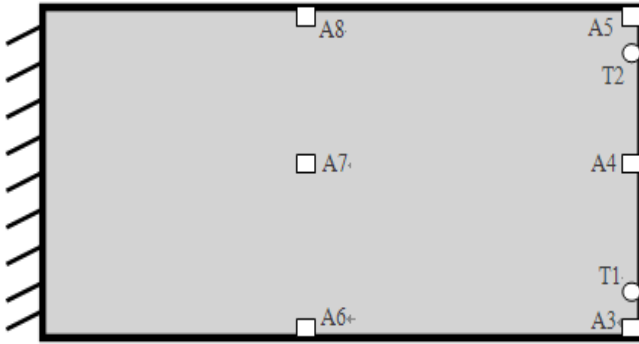


Fig. 10. Aluminum sheet and the sensor placement

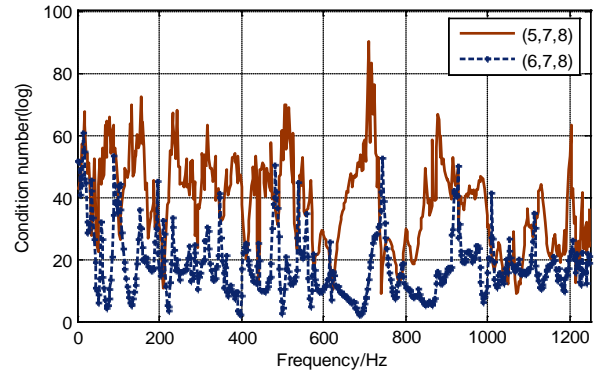


Fig. 11. Comparison of condition number

In Fig. 11, "----" represents the condition number of $H_{m \times l}$ for combination (A6, A7, A8); "___" represents the condition number of $H_{m \times l}$ for combination (A5, A7, A8).

As shown in Fig.11, response combination (A5, A7, A8) has lower condition number than the other one at the concerned frequency range, and the combination (A5, A7, A8) should have a better identification result than the other one. The identification results of the experiment are shown in the Fig. 12 to Fig. 15.

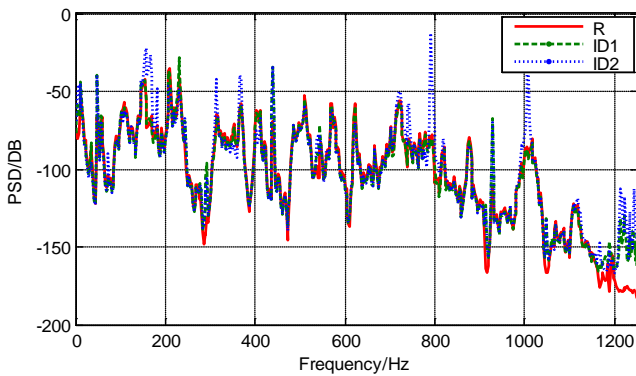


Fig. 12. Identification of T1 using (A5, A7, A8)

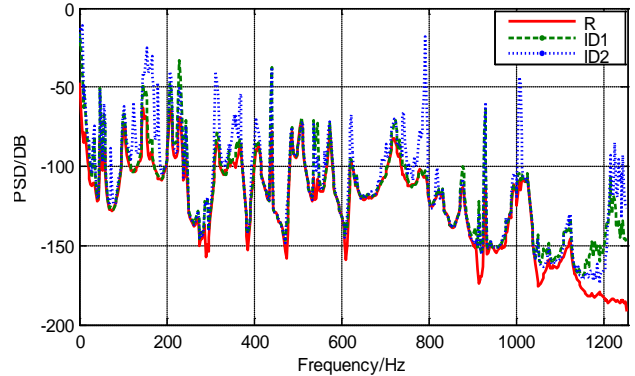


Fig. 13. Identification of T2 using (A5, A7, A8)

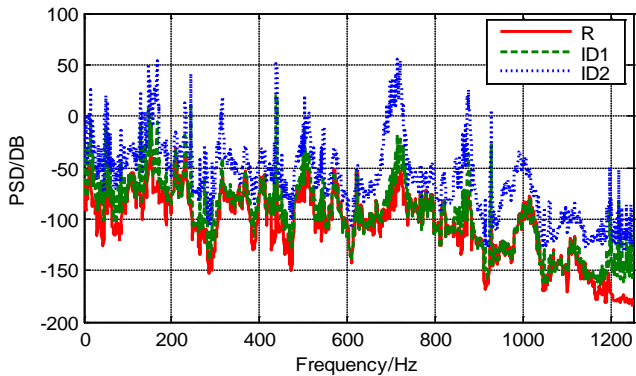


Fig. 14. Identification of T1 using (A6, A7, A8)

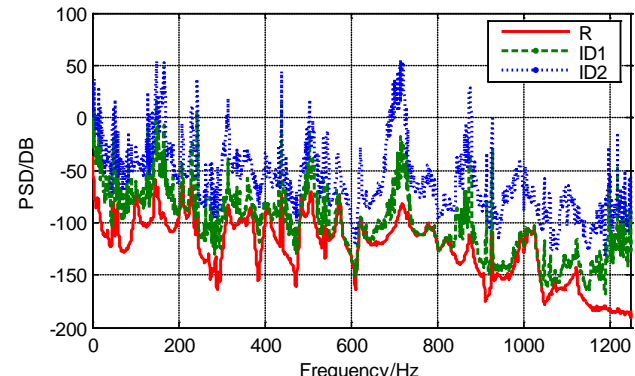


Fig. 15. Identification of T2 using (A6, A7, A8)

In Fig. 12~15, "R" represents the real value of the random load; "ID1" represents the identification result by CNWAM; "ID2" represents the identification result by DIM.

As shown in Fig.12-15, the proposed approach can obtain a better identification result when using the same response combination. It proves condition number weighted average method to be effective. A better identification result is obtained by the response combination (A5, A7, A8) than (A6, A7, A8), especially, nearby the natural frequencies. There is only one different measure point between the two combinations, and even a skilled engineer cannot easily to get a clear judgment of which one would be better. These errors are caused by the system, the algorithm cannot remove it completely. While, the selection of appropriate response measure points can change the system frequency response function H_{msl} and improve the identification precise to some extent. The condition number of FRF can provide reference for experimenter to choose a better response combination. A figure as Fig. 11 can help the engineers to select combination at the frequency range they care about before identification process. This is the effective way to improve the identification precision and to save workload.

On the whole, CNWAM can effectively improve the identification accuracy (Curve ID1 in Fig.12 to Fig. 15) relative to the direct inverse method (Curve ID2 in Fig.12 to Fig. 15). The errors are unacceptable using DIM, especially if the measure point is selected improperly (Curve ID2 in Fig.14 and Fig. 15).

In addition, according to the author's experiences, The measuring point selection should also follow some principles as follows:1. Response measure points should avoid being set on modal shape node, should as far as possible be set close by the peak or valley of modal shape. 2. The measure points should avoid being set close by each other in the same direction. 3. The measure points should be set on the places where the vibration amplitudes are large enough, so that the signal-to-noise ratio would be improved. 4. If it is possible, greater system damping can improve the identification accuracy.

5. Conclusions

For complex engineering structures random loading identifications, direct inverse of FPF gets bad accuracy identification result. The condition number weighted average method is presented in this paper to improve the identification precision. Simulation and experiment are both prove its effectiveness. The selection of the response measure points is one of the key points to improve the loading identification accuracy. Condition number of the FRF is used in the paper to select the appropriate combination of the response measure points. The experimental results demonstrated its effectiveness.

However, more work remains to be done, e.g. the problem of noise disturbance should be extensively investigated in a stricter way and the benefits could be investigated of using Kalman or H1 filtering techniques to further improve the identification precision.

6. Acknowledgement

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