Interval identification of thermal parameters for convection-diffusion heat transfer problems

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Abstract

This paper presents a numerical method to identify the intervals of thermal parameters for steady state convection-diffusion heat transfer problems when uncertainty of measurement is characterized by the interval. A two step strategy is suggested to estimate the lower and upper bounds of thermal parameters in the terms of central value and radius. A 2D numerical example is provided to verify the proposed approach.

Keywords: interval; identification; uncertainty; estimate; thermal parameters.

1. Introduction

Thermal parameter estimation is an important issue related with many engineering aspects (Rodriguez and Nicolau,2012;Liu and Xu,2000;Reverberi and Fabiano,2013), and is usually based on an assumption that measurement is deterministic. Unfortunately all measurements are indeterministic, and contain various uncertainties(Hugh and Steele,1999). Such uncertainties may result in fault results in the analysis process, and necessitate to take into account(Wang and Qiu,2010;Shome,1999).The point is how to estimate the uncertainty of thermal parameter caused by the measurement uncertainty.

There are mainly three mathematical means to describe the uncertainty, including probabilistic method, fuzzy theory, and interval analysis(Rao and Berke,1997; Elishakoff,1998;Muhanna and Mullen,2001). The interval analysis does not require any prior knowledge or assumption of the uncertainty distribution inside their definition ranges as probability and fuzziness do, only the bounds of changes of the uncertain quantities are necessary(Jiang, Liu and Han,2008). As matter of fact, there were some previous reports concerned with the interval analysis in forward heat transfer problems. C.P. Sebastiao(pereiar,2004) presented an interval uncertainty assessment in 1-D thermal basin modeling via an Element-By-Element(EBE) technique, and provided good results in accordance with those given by the Mento Carlo and combinatorial methods. J.P. Li(Li and Chen,2009)employed a perturbation technique to acquire the lower and upper bounds of temperatures for the transient heat conduction problem with interval parameters. H.T. Yang(Xue and Yang,2013)developed two methods to estimate the bounds of temperatures by utilizing Taylor and Neumann expansion skills for convection-diffusion heat transfer problems when thermal parameters are uncertain and described by intervals. However by authors best knowledge, there seems no any report directly related to the interval estimation of thermal parameters for uncertain convection-diffusion heat transfer problems so far, although great achievement has been gained for the interval estimation of physical parameters in structural engineering(Wang and Qiu,2010;Jiang, Liu and Han,2008) and other aspects(Braems and Berthier,2000;Jorge,2004; Sergey and Nazin,2005).

This paper focuses on the interval estimation of thermal parameters for the steady state conduction-diffusion heat transfer problem when the uncertainty of temperature measurements is characterized by intervals. Section 2 gives a brief description of numerical modeling of forward convection-diffusion heat transfer problems with interval parameters; Section 3 presents a two step strategy to estimate interval bounds of thermal parameters when measurement temperature is characterized by interval; Section 4 provides 2D numerical tests to verify the proposed approach; Section 5 reaches the conclusion.

2. Numerical modeling of forward convection-diffusion heat transfer problems with interval parameters

The governing equation of steady state convection-diffusion heat transfer problems is(Platten and Legos,1984)

$$cu_i T_{,i} = \left[k_{ij} T_{,j} \right]_{,i} + Q \quad \boldsymbol{x}_i \subset \Omega \tag{1}$$

where T stands for the temperature, c and k_{ij} are heat capacity and thermal conductivity respectively, u_i refers to the vector of the fluid velocity, Q is a volumetric heat source term, $\boldsymbol{x}_i$ is the vector of the coordinates, Ω represents the space domain of the problem and subscript i, j refers to a summation index($i, j = 1,2$ for the 2D problem, $i, j = 1,2,3$ for the 3D problem).

The boundary condition is given by

$$T = T_B \quad x_i \in \Gamma_1 \tag{2}$$

$$n_i k_{ij} T_{,j} = q \quad x_i \in \Gamma_2 \tag{3}$$

where T_B and q are prescribed functions, $\Gamma = \Gamma_1 + \Gamma_2$ represents the whole boundary of Ω, and $\boldsymbol{n}_i$ refers to the outward unit normal along Γ_2.

Eqs. (1-3) can be formulated in a FEM form(Huebner and Thornton,1995)

$$\boldsymbol{KT} = \boldsymbol{P} \tag{4}$$

where $\boldsymbol{T}$ refers to the general nodal vector of temperature.

$$\boldsymbol{K} = \sum_e \int_{\Omega^e} \boldsymbol{N}_{,i}^{\mathrm{T}} k_{ij} \boldsymbol{N}_{,i} d\Omega^e + \sum_e \int_{\Omega^e} \boldsymbol{N}^{\mathrm{T}} cu_i \boldsymbol{N}_{,i} d\Omega^e \tag{5}$$

$$\boldsymbol{P} = \sum_e \int_{\Omega^e} \boldsymbol{N}^{\mathrm{T}} \boldsymbol{Q} d\Omega^e + \sum_e \int_{\Gamma^e} \boldsymbol{N}^{\mathrm{T}} \boldsymbol{q} d\Gamma^e \tag{6}$$

$\boldsymbol{N}$ stands for a matrix of shape functions.

Assume the thermal parameter vector $\boldsymbol{b}=\left[k_{ij}, u_i, c, Q, q\right]^T$ is an interval vector and is described by(Alefeid and Herzberger,1983;Moore,1979).

$$\boldsymbol{b}^I = \left[\underline{\boldsymbol{b}}, \overline{\boldsymbol{b}}\right] = \boldsymbol{b}^c + \Delta\boldsymbol{b} \cdot e_\Delta \tag{7}$$

$$\boldsymbol{b}^c = \left(\overline{\boldsymbol{b}} + \underline{\boldsymbol{b}}\right)/2 \quad \Delta\boldsymbol{b} = \left(\overline{\boldsymbol{b}} - \underline{\boldsymbol{b}}\right)/2 \quad e_\Delta = [-1,1] \tag{8}$$

where $\boldsymbol{b}^c$ and $\Delta\boldsymbol{b}$ represent the vectors of central value and radius of $\boldsymbol{b}^I$, $\underline{\boldsymbol{b}}$ and $\overline{\boldsymbol{b}}$ refer to the lower and upper bounds vectors of $\boldsymbol{b}^I$.

Near the neighbor of $\boldsymbol{b}^c$, $\boldsymbol{b}$ can be described by

$$\boldsymbol{b} = \boldsymbol{b}^c + \delta\boldsymbol{b},\ \delta\boldsymbol{b} \in \Delta\boldsymbol{b}^I = [-\Delta\boldsymbol{b}, \Delta\boldsymbol{b}] \tag{9}$$

Utilizing the Taylor series expansion, the first order approximation of the solution of Eq. (4) can be written as(Qiu and Wang,2004)

$$\boldsymbol{T}(\boldsymbol{b}) = \boldsymbol{T}(\boldsymbol{b}^c + \delta\boldsymbol{b}) = \boldsymbol{T}^c + \sum_{j=1}^{m} \frac{\partial \boldsymbol{T}(\boldsymbol{b}^c)}{\partial b_j}\delta b_j \tag{10}$$

where

$$\boldsymbol{T}^c = \boldsymbol{T}(\boldsymbol{b}^c) \tag{11}$$

$$\frac{\partial \boldsymbol{T}(\boldsymbol{b}^c)}{\partial b_j} = \left[\boldsymbol{K}(\boldsymbol{b}^c)\right]^{-1}\left[\frac{\partial \boldsymbol{P}(\boldsymbol{b}^c)}{\partial b_j} - \frac{\partial \boldsymbol{K}\left(\boldsymbol{b}^c\right)}{\partial b_j}\boldsymbol{T}^c\right] \tag{12}$$

The lower and upper bounds of $\boldsymbol{T}^I$ are estimated by

$$\underline{\boldsymbol{T}}(\boldsymbol{b}) = \boldsymbol{T}^c - \Delta\boldsymbol{T} = \boldsymbol{T}\left(\boldsymbol{b}^c\right) - \sum_{j=1}^{m}\left|\left[\boldsymbol{K}(\boldsymbol{b}^c)\right]^{-1}\left[\frac{\partial \boldsymbol{P}(\boldsymbol{b}^c)}{\partial b_j} - \frac{\partial \boldsymbol{K}\left(\boldsymbol{b}^c\right)}{\partial b_j}\boldsymbol{T}^c\right]\right|\Delta b_j \tag{13}$$

$$\overline{\boldsymbol{T}}(\boldsymbol{b}) = \boldsymbol{T}^c + \Delta\boldsymbol{T} = \boldsymbol{T}\left(\boldsymbol{b}^c\right) + \sum_{j=1}^{m}\left|\left[\boldsymbol{K}(\boldsymbol{b}^c)\right]^{-1}\left[\frac{\partial \boldsymbol{P}(\boldsymbol{b}^c)}{\partial b_j} - \frac{\partial \boldsymbol{K}\left(\boldsymbol{b}^c\right)}{\partial b_j}\boldsymbol{T}^c\right]\right|\Delta b_j \tag{14}$$

where

$$\Delta\boldsymbol{T} = \sum_{j=1}^{m}\left|\left[\boldsymbol{K}(\boldsymbol{b}^c)\right]^{-1}\left[\frac{\partial \boldsymbol{P}(\boldsymbol{b}^c)}{\partial b_j} - \frac{\partial \boldsymbol{K}\left(\boldsymbol{b}^c\right)}{\partial b_j}\boldsymbol{T}^c\right]\right|\Delta b_j \tag{15}$$

utilizing Eqs. (11) and (15), lower and upper bounds of temperatures can be estimated for the convection-diffusion heat conduction problem with interval parameters.

3. Interval identification of thermal parameters

Assume that the measurement temperature T^* is characterized by the interval via $\left[\underline{\boldsymbol{T}}^*, \overline{\boldsymbol{T}}^*\right]$ where $\underline{\boldsymbol{T}}^*$ and $\overline{\boldsymbol{T}}^*$ stand for lower and upper bounds of T^*. The central value and radius of T^* are given by

$$\boldsymbol{T}^{c*} = \frac{\overline{\boldsymbol{T}}^* + \underline{\boldsymbol{T}}^*}{2}, \quad \Delta \boldsymbol{T}^* = \frac{\overline{\boldsymbol{T}}^* - \underline{\boldsymbol{T}}^*}{2} \tag{16}$$

Using $\underline{\boldsymbol{T}}^*$ and $\overline{\boldsymbol{T}}^*$, the interval estimation of thermal parameters can be realized via the identification of $\boldsymbol{b}^c$ and $\Delta \boldsymbol{b}$.

The identification of $\boldsymbol{b}^c$ and $\Delta \boldsymbol{b}$ is conducted by minimizing two L_2 norms defined by

$$\left[\boldsymbol{L}\boldsymbol{T}^c\left(\boldsymbol{b}^c\right) - \boldsymbol{T}^{c*}\right]^2 \tag{17}$$

$$\left[\boldsymbol{L}\Delta\boldsymbol{T}\left(\Delta\boldsymbol{b}\right) - \Delta\boldsymbol{T}^*\right]^2 \tag{18}$$

where $\boldsymbol{T}^c$ and $\Delta \boldsymbol{T}$ is given by Eqs. (11) and (15), $\boldsymbol{L}$ is a matrix mapping the relationship of $\boldsymbol{T}$ and the vector of measurement points.

The above minimizations can be realized by the L-M (Levenberg—Marquardt) algorithm(Levenberg,1994). For a problem defined by

$$\mathrm{Min} \prod = \frac{1}{2}\boldsymbol{F}^{\boldsymbol{T}}(\boldsymbol{\varphi})\boldsymbol{F}(\boldsymbol{\varphi}) \tag{19}$$

the major procedure of L-M algorithm includes

$$\boldsymbol{\varphi}^{\boldsymbol{m+1}} = \boldsymbol{\varphi}^{\boldsymbol{m}} + \Delta\boldsymbol{\varphi} \tag{20}$$

$$\left(\boldsymbol{G}^{\boldsymbol{T}}\boldsymbol{G} + \mu^n \boldsymbol{I}\right)\Delta\varphi = -\boldsymbol{G}^{\boldsymbol{T}}\boldsymbol{F} \tag{21}$$

where $\boldsymbol{G}^{\boldsymbol{T}}$ stands for the gradient matrix of $\boldsymbol{F}$ with respect to $\boldsymbol{\varphi}$, μ is an non-negative damping factor, and $\boldsymbol{I}$ refers to an identity matrix.

When $\|\Delta\boldsymbol{\varphi}\| \le \beta$, the above iteration stops, β refers to an error tolerance.

For Eq. (17)

$$\boldsymbol{\varphi} = \boldsymbol{b}^c \tag{22}$$

$$G = L\frac{\partial T^c}{\partial b^c} = L\sum_{j=1}^{m} K^{-1}\left(\frac{\partial P}{\partial b_j} - \frac{\partial K}{\partial b_j} T^c\right) \tag{23}$$

$$F = LT^c - T^{c*} \tag{24}$$

For Eq.(18)

$$\varphi = \Delta b \tag{25}$$

$$G = L\frac{\partial \Delta T}{\partial \Delta b} = L\sum_{j=1}^{m} \left| K^{-1}\left[\frac{\partial P}{\partial b_j} - \frac{\partial K}{\partial b_j} T^c\right]\right| \tag{26}$$

$$F = L\Delta T - \Delta T^* \tag{27}$$

Therefore the lower and upper bounds of thermal parameters are given by

$$\underline{b} = b^c - \Delta b \tag{28}$$

$$\overline{b} = b^c + \Delta b \tag{29}$$

4. Numerical verification

For the simplicity, all the computing parameters are assumed dimensionless.

Consider an inverse 2D steady state convection-diffusion heat transfer problem in a 10 × 10 rectangular domain which is meshed by 10×10 finite elements, as shown in Fig.1.

The boundary condition is defined by

$$T(x, y = 0) = 1,\ T(x, y = 10) = 0,$$
$$T(x = 0, y)\big|_{y\in[0,4]} = 1,\ T(x = 0, y)\big|_{y\in(4,10]} = 0,\ T(x = 10, y) = 0$$

The effect of noisy data is taken into account in the form

$$\underline{T}_n^* = (1 + \sigma\varsigma)\underline{T}^* \tag{30}$$

$$\overline{T}_n^* = (1 + \sigma\varsigma)\overline{T}^* \tag{31}$$

where $\underline{\boldsymbol{T}}_n^*$ and $\overline{\boldsymbol{T}}_n^*$ represents the vectors of lower and upper bounds of measured or stimulated temperature containing the noisy data, ς is a random variable between -0.5~0.5, σ refers to a noisy level.

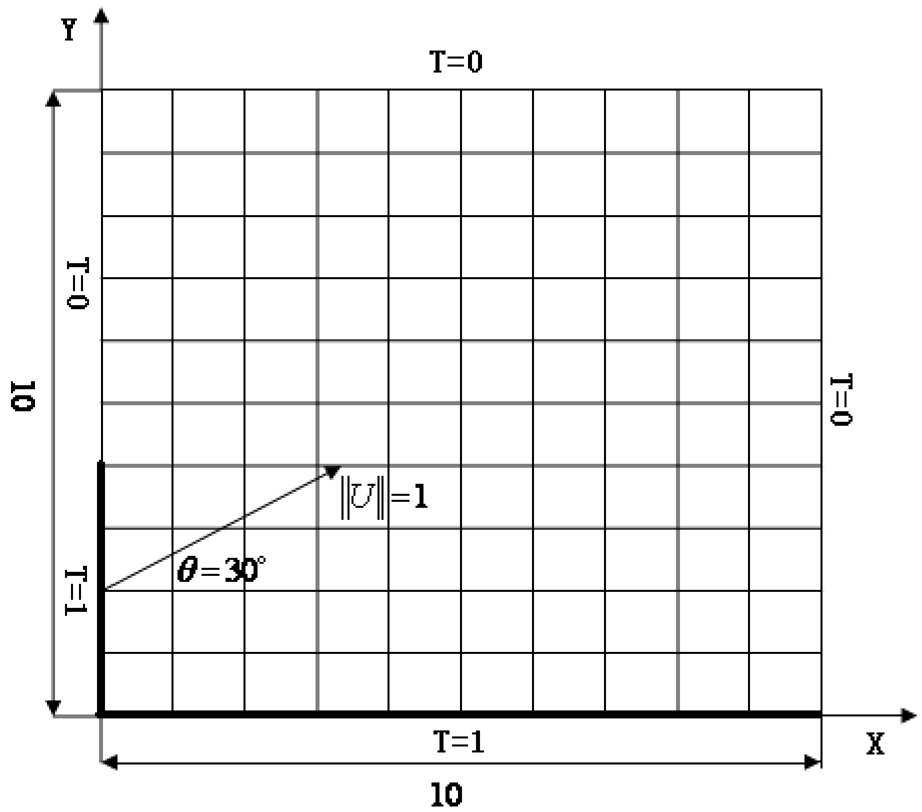


Figure 1 The FE mesh of the rectangular plate

Case 1

$c^c = 1$, $\Delta c = \alpha \cdot c^c$, $u_x^c = \cos 30°$, $\Delta u_x = \alpha \cdot u_x^c$, $u_y^c = \sin 30°$, $\Delta u_y = \alpha \cdot u_y^c$, α is defined as the degree of uncertainty. The intervals of k_{xx} and k_{yy} are to be identified.

Tab. 1 exhibits the solutions with different initial guesses; Tab. 2 presents solutions with different arrangement of measuring points as shown in Fig2; Tab. 3 gives solutions at different noisy levels.

Tab. 1. The effects of initial guesses on the results

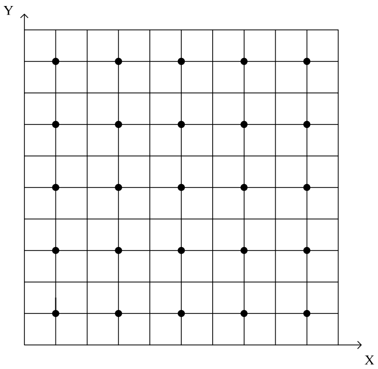
Identified parameters	1			2			Actual values
	Initial guesses	Results of identification	Iterative steps	Initial guesses	Results of identification	Iterative steps	
k_{xx}^c	0.1	0.6	6	5	0.6	9	0.6
Δk_{xx}	0.1	0.06	4	5	0.06	4	0.06
k_{yy}^c	0.1	0.5	6	5	0.5	9	0.5
Δk_{yy}	0.1	0.05	4	5	0.05	4	0.05

Tab. 2 The effect of different distribution of measuring points on the results

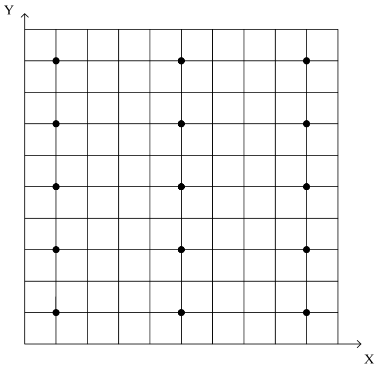
Identified parameters	Results of identification			Actual values
	25 measuring points	15 measuring points	10 measuring points	
k_{xx}^{c}	0.6	0.6	0.6	0.6
Δk_{xx}	0.06	0.06	0.06	0.06
k_{yy}^{c}	0.5	0.5	0.5	0.5
Δk_{yy}	0.05	0.05	0.05	0.05

Tab. 3. The effects of noisy data on the results

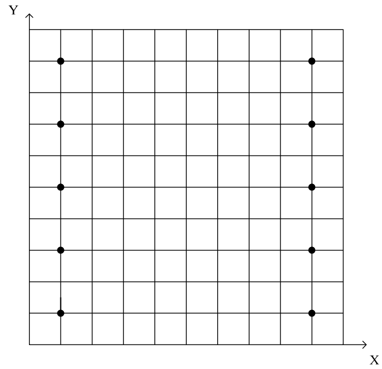
Identified parameters	$\sigma = 1\%$		$\sigma = 5\%$		Actual values
	Result	Error(%)	Result	Error(%)	
k_{xx}^{c}	0.6041	0.68	0.5913	1.45	0.6
Δk_{xx}	0.0603	0.50	0.0593	1.17	0.06
k_{yy}^{c}	0.5078	1.56	0.4833	3.34	0.5
Δk_{yy}	0.0498	0.40	0.0504	0.80	0.05



(a) 25 measuring points



(b) 15 measuring points



(c) 10 measuring points

Fig 2 Distribution of measuring points

Case 2:

$c^c = 1$, $\Delta c = \alpha \cdot c^c$, $k_{xx}^c = 0.6$, $\Delta k_{xx} = \alpha \cdot k_{xx}$, $k_{yy}^c = 0.6$, $\Delta k_{yy} = \alpha \cdot k_{yy}$. The intervals of u_x and u_y are to be identified.

Tab. 4 and 5 exhibit solutions with different initial guesses and different arrangement of measuring points, Tab. 6 gives solutions at different noisy levels.

Numerical tests indicate

1. Initial guesses seems no impact on the solution, as shown in Tab. 1 and Tab. 4 where the largest ratio between initial guesses and true values is 10. The accurate intervals were identified with few iterative steps which shows the high efficiency of the presented method.
2. Since the problem defined in the numerical test is homogeneous the variation of number and location of measuring points gives no impact on the solution, as shown in Tab. 2 and Tab.5, respectively.
3. The proposed algorithm is not sensitive to noisy data when $\sigma \le 5\%$, the maximum relative error is 3.34% when $\sigma = 5\%$.
4. Only few iterations are required to obtain satisfactory results, as shown in Tab. 1 and Tab.4.

Tab. 4. The effects of initial guesses on the results

Identified parameters	1			2			Actual values
	Initial guesses	Results of identification	Iterative steps	Initial guesses	Results of identification	Iterative steps	
u_x^c	0.1	0.866	12	5	0.866	6	0.866
Δu_x	0.1	0.086	4	5	0.086	4	0.086
u_y^c	0.1	0.5	12	5	0.5	6	0.5
Δu_y	0.1	0.05	4	5	0.05	4	0.05

Tab. 5 The effect of different distribution of measuring points on the results

Identified parameters	Results of identification			Actual values
	25 measuring points	15 measuring points	10 measuring points	
u_x^c	0.866	0.866	0.866	0.866
Δu_x	0.086	0.086	0.086	0.086
u_y^c	0.5	0.5	0.5	0.5

Δu_y	0.05	0.05	0.05	0.05

Tab. 6. The effects of noisy data on the results

Identified parameters	$\sigma = 1\%$		$\sigma = 5\%$		Actual values
	Result	Error(%)	Result	Error(%)	
u_x^c	0.8714	0.54	0.8517	1.65	0.866
Δu_x	0.0869	1.05	0.0856	0.47	0.086
u_y^c	0.5041	0.82	0.4892	2.16	0.5
Δu_y	0.0504	0.80	0.0489	2.2	0.05

Conclusions

Since there seems no direct report concerned with the inverse convection-diffusion heat transfer problems when the uncertainty of temperature measurements is characterized by the interval, this paper attempts to present a numerical model to solve this kind of problem. In terms of central value and radius of interval, a two step strategy is suggested to estimate the lower and upper bounds of thermal parameters and fluid velocity. The L-M method is employed in the estimation procedure, and a numerical test is given to illustrate the advantages of the proposed algorithm with the consideration of initial guess, arrangement of measurement points, and data noise.

The presented model is verified via some numerical tests, however due to the lack of actual uncertainty information either from experiment or industry (some of parameter is based on assumption) more efforts for the further model V&V are required. For the model application of industry, in addition to the numerical verification similar to this paper, the experiment based verification is particularly required. On the other hand we need collect sufficient message on the interval uncertainty from industry, such as the width of interval, noisy level, etc., and validate/verify the proposed model via some industry cases to secure the validity of identification results.

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