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A hybrid approach to structural topology optimization of vehicle for crashworthiness

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Abstract

Structural topology optimization of vehicle for crashworthiness is a challenging subject because it involves geometric and material nonlinearity as well as transit dynamic analysis. The difficulty of gradient calculation in crashworthiness design problem constitutes a major obstacle to apply the existing topology optimization methods such as homogenization method and SIMP method. HCA (hybrid cellular automata) approach is one of heuristic approaches and does not need gradient information to get a crashworthiness topology optimum design. Each execution of HCA iteration needs a complete crashworthiness analysis to obtain the yield parameters for the update rule of design variables, which renders the method high computational cost. The well-known inertia relief method replaces the transit dynamic analysis by approximate static analysis under the impact load and inertia load. However, in car collision event the magnitude and spatial distribution of the impact load is unknown in prior and depends on the structure being crashed. The present study proposes a hybrid approach by integrating the improved inertia relief method with HCA. The structural topology optimization is obtained iteratively under the given impact load with the inertia relief method, nonlinear static analysis and HCA approach. A simplified car model example is tested to show the effectiveness and efficiency of the hybrid approach.

Keywords: Crashworthiness, Energy absorption, Inertia relief, Topology Optimization

Introduction

Crashworthiness design is an important and challenging subject for vehicle design. The performance of vehicle in a crash event has a direct impact on the safety of drivers, passengers and pedestrians. The difficulty of crashworthiness analysis is widely recognized, because it involves highly nonlinear and dynamic effects of the vehicle structure, such as large deformation, large strain, material nonlinearity, contact, structural and material damage and even non-linear wave propagation. The formulation of crashworthiness optimization problem should both concern the criterion of auto industry and the product cost, so energy absorption and mass cost should both be reflected in the optimization formulation.

Over the decades, many contributions have been done to enhance the vehicle crashworthiness and prevent the passenger and driver from injuries. While most works optimize one vehicle component, a few works study topology optimization (Bendsøe, M.P., 2003) of the whole car body (Mayer et al., 1996; Pedersen, 2003; Soto, 2004). Mayer et al. In structural topology optimization, there are two methods most widely used, homogenization method and SIMP (Solid Isotropic Material with Penalization) method. But both of them are gradient-based approach, they can't be used in nonlinear problem directly. The HCA method does not use mathematical programming techniques, but makes use of the cellular automata paradigm to drive the design synthesis. This approach is inspired by the biological process of bone remodeling and was first presented by Tovar (2004). Patel and Tovar (2009) have applied the HCA method to the crashworthiness topology optimization problem, which considered both material and geometrically nonlinear behavior. In HCA method, each step in optimization process requires a complete transit nonlinear analysis, which is computationally very expensive. The inertia relief method (Nelson et al., 1977; Lin liao, 2011) is an approximate analytical technique for transit analysis. It has been applied by the aerospace industry in structural analysis and optimization of free-free vehicle for many years.

In this paper, a new hybrid approach, which integrates the improved inertia relief method with HCA method, is developed to vehicle energy absorption optimization. The results show that the hybrid approach could reduce the computational cost of the optimization significantly and get a better design of the crashworthiness design problem.

1 Modified Inertia relief method and its accuracy analysis

The basic idea of inertia relief assumes that a free-free structure under impact load moves as a rigid body. In order to use the inertia relief method, the impact load should be first determined and applied on a totally unconstraint structure, and the accelerations is calculated by rigid body moving laws.

Figure 1. The simple spring-mass model

A simple spring-mass model in Figure 1 illustrates the process that using inertia relief to get an approximate result. The mass-spring system consists of two masses M_1 and M_2 connected by a spring of stiffness K and is subject to a dynamic external force F(t) on mass M_1. First, the model is treated as a rigid body of total mass M_1+M_2. The acceleration of the system can be obtained as:

$$a_{sys}(t)=F(t)/(M_1+M_2) \tag{1}$$

Accelerations of M_1 and M_2 are equal to the system acceleration and the inertia forces $-M_1a_{sys}$, $-M_2a_{sys}$ act on two masses, respectively. Static analysis under the external load F(t) and the inertia forces $-M_1a_{sys}$, $-M_2a_{sys}$ results the internal force of the spring K:

$$F_K(t)=F(t)M_2/(M_1+M_2) \tag{2}$$

But this model is not fully suitable for a crash event because the magnitude and its spatial distribution of the impact load form is not known in prior and is dependent on the structure being crashed. In this paper, to improve the inertia relief method and study its accuracy, we propose an improved simple dynamic analysis model which was presented by Wu and Yu (2001). The improved model is shown in figure 2. Since in car crash event, there exists often a lower stiffness part in the front part of the structural body, we assume the stiffness of K_1 is less than K_2.

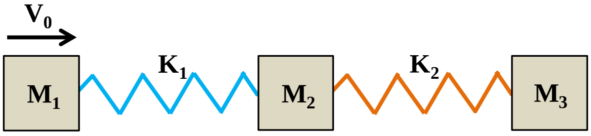


Figure 2. The simple dynamic model presented by Wu and Yu

In comparison with former model, the new model is built by adding one moving mass and one spring in the original model, the latter of which simulates the structural behavior of the local impact area or the lower stiffness part. As the moving mass M_1 with initial velocity V_0 impacts the mass M_2, the spring K_1 is compressed and the whole mass-spring system moves as an integrated one. At certain stage, the moving mass M_1 will be rebounded. However, here we are only interested in their behavior before rebound happens. By transit dynamic analysis of this model, the analytical forms of the internal forces of the two springs can be expressed as:

$$\begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} = \begin{Bmatrix} k_1 \\ k_2 r_1 \end{Bmatrix} \frac{V_0 r_2}{\omega_1^*(r_1 - r_2)} \sin(\omega_1^* t) - \begin{Bmatrix} k_1 \\ k_2 r_2 \end{Bmatrix} \frac{V_0 r_1}{\omega_2^*(r_1 - r_2)} \sin(\omega_2^* t) \tag{3}$$

Where

$$\omega_1^2 = \frac{k_1(M_1 + M_2)}{M_1 M_2} \qquad \omega_2^2 = \frac{k_2(M_2 + M_3)}{M_2 M_3}$$

$$\alpha_1 = \frac{M_1}{M_1 + M_2} \qquad \alpha_2 = \frac{M_3}{M_2 + M_3}$$

$$r_1 = \frac{\omega_1^2 - \omega_1^{*2}}{\alpha_2 \omega_2^2} = \frac{\alpha_1 \omega_1^2}{\omega_2^2 - \omega_1^{*2}} \qquad r_2 = \frac{\omega_1^2 - \omega_2^{*2}}{\alpha_2 \omega_2^2} = \frac{\alpha_1 \omega_1^2}{\omega_2^2 - \omega_2^{*2}}$$

And the first and second frequency of the mass-spring system is given by

$$\omega_{12}^{*2} = \frac{\omega_1^2 + \omega_2^2}{2} \pm \frac{1}{2}\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\alpha_1\alpha_2\omega_1^2\omega_2^2}$$

Through the analytical forms, we find that when $\omega_1 << \omega_2$ the analytical forms could be simplified, and the peak load could be express as:

$$F_1(t) = -k_1 \frac{V_0}{\alpha_2 \omega_1^*(r_1 - r_2)} \sin(\omega_1^* t) \tag{4}$$

$$F_2(t) = -k_2 \frac{\alpha_1 \omega_1^2 V_0}{\alpha_2 \omega_1^*(r_1 - r_2)\omega_2^2} \sin(\omega_1^* t) - k_2 \frac{\alpha_1 V_0 \omega_1^2}{\alpha_2 \omega_2^*(r_1 - r_2)\omega_2^2} \sin(\omega_2^* t) \tag{5}$$

When $\sin(\omega_1^* t) = 1$, the internal force of spring K_1 reaches its maximum, the internal force of the spring K_2 by the inertia relief method at the same time could be expressed as:

$$F_2^{ir} = \alpha_2 F_1^t \tag{6}$$

And the analytical form of the internal forces of spring K_2 could be further simplified as following:

$$F_2^t = -k_2 \frac{\alpha_1 \omega_1^2 V_0}{\alpha_2 \omega_1^*(r_1 - r_2)\omega_2^2} - k_2 \frac{\alpha_1 V_0 \omega_1^2}{\alpha_2 \omega_2^*(r_1 - r_2)\omega_2^2} \sin\left(\frac{\omega_2^* \pi}{\omega_1^* 2}\right) \tag{7}$$

Then we could get:

$$F_2^t = \alpha_2 \left(1 + \frac{\omega_1^*}{\omega_2^*} \sin\left(\frac{\omega_2^* \pi}{\omega_1^* 2}\right)\right) F_1^t \tag{8}$$

$$\left|\frac{F_2^t - F_2^{ir}}{F_2^{ir}}\right| = \frac{\omega_1^*}{\omega_2^*} \left|\sin\left(\frac{\omega_2^* \pi}{\omega_1^* 2}\right)\right| \le \frac{\omega_1^*}{\omega_2^*} \tag{9}$$

The error of maximum force of the inertia relief's result and the analytical result could be estimated by the ratio of the first order and second order frequencies.

2 Numerical examples of the modified inertia relief method

A numerical example has been carried out to verify the above conclusion obtained from the new mass-spring model. We choose seven parameter sets of the spring-mass model for comparison. The first model has parameters M_1=50Kg, M_2=50Kg, M_3=50Kg, K_1=2000N/m, K_2=2000N/m, and the other six sets just changes the stiffness of spring K_2 by 5times, 10 times, 50 times, 100times, 500times and 1000 times, respectively. The results of the seven models are listed in table 1. Here, the internal force results are chosen at the time when the internal force of spring K_1 reaches its maximum.

Table 1. The results of inertia relief method and dynamic analysis

	K_2	$F_2^{t_c}$	F_2^{ir}	$\left\|\frac{F_2^{t_c}-F_2^{ir}}{F_2^{ir}}\right\|$	$\frac{\omega_1^*}{\omega_2^*}$
First model	2.00E+03	-304.10	-569.07	0.466	0.577
K2*5	1.00E+04	-954.75	-608.00	0.570	0.366
K2*10	2.00E+04	-739.27	-633.99	0.166	0.267
K2*50	1.00E+05	-625.12	-643.94	0.029	0.122
K2*100	2.00E+05	-683.79	-644.57	0.061	0.086
K2*500	1.00E+06	-635.87	-645.32	0.015	0.038
K2*1000	2.00E+06	-638.23	-645.40	0.011	0.027

The results showed in table 1 improve that the difference between the results of two methods will be smaller when the ratio between the first and second order frequencies decreaseing. And the difference is smaller than the ratio of frequencies when spring K_2's stiffness larger than 10 times of spring K_1's stiffness which verify the previous conclusion. We can also change the other parameters to decrease the differnece, and this part will be showed in further work.

3 Hybrid Cellular Automaton Algorithm

The information of each element contains two parts of parameters, one is the design variable x_i, and the other is the yield parameter S_i gained by FEM. In HCA method, the yield parameters in its neighborhood are used to modify the yield parameter in the concerned cell. There are a variety of neighborhood forms, the form used in this paper is the right one shown in figure 3, and the yield parameter can be expressed as:

$$\bar{S}_i^{(k)} = \frac{S_i^{(k)} + \sum_{n \in N(i)} S_n^{(k)}}{N+1} \tag{10}$$

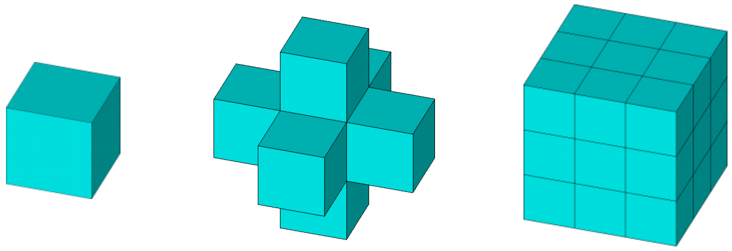


Figure 3. Three neighborhood forms

Due to the consideration of both requirement of industry criterion and product cost, we set the energy absorption maximization as the objective and the uniform strain energy distribution as the criterion. The structural mass factor is constrained, as Patel al did in his work. The mathematical formulation is

$$\begin{aligned} \min_{x} \quad & \sum_{i=1}^{N} \left| \bar{S}_i - S^* \right| \\ \text{Such that} \quad & \sum_{i=1}^{N} x_i \rho v_i = M_f^* * \sum_{i=1}^{N} \rho v_i \\ & \mathbf{KU} = \mathbf{P} \\ & x_{\min} \le x_i \le 1 \end{aligned} \quad (11)$$

In this paper, the update rules and yield parameter of HCA method are same as Patel did. S_i is equal to U_i, the strain energy of the ith element. The $S^{*(k)}$ is the global set point value of the *k*th iteration, by adjusting $S^{*(k)}$ from iteration to iteration the mass fraction constraint M_f^* is gradually imposed. The update rule of the design parameter x_i to make the strain energy uniform can be expressed as:

$$\Delta x_i = max\left\{-0.1, min\left\{K_P\left(\bar{S}_i^{(k)} - S^{*(k)}\right), 0.1\right\}\right\} \quad (12)$$

The K_p is a problem-dependent constant, and the $S^{*(k)}$ is a global set point. The material follows the bilinear form, and parameterization of the material is expressed as:

$$E_i(x_i) = x_i^3 E_0 \qquad \sigma_{Yi}(x_i) = x_i^3 \sigma_{Y0} \qquad E_{hi}(x_i) = x_i^3 E_{h0} \quad (13)$$

4 A new hybrid method of HCA and IRM

The new integrated approach is described as follows:

Step 1. Define the design domain, materials properties, initial velocity v_0, and assume the initial design $x^{(0)}$ variables distribution, and specify the convergence condition the tolerance of the peak load change ΔP_t.

Step 2. Do a complete transient dynamic analysis, and find the peak value and its spatial distribution of the impact load of the design area.

Step 3. Apply the peak load to the design and carry out a static inertia relief analysis, which is a static nonlinear analysis under the peak load and inertia loads. The error of the results between inertia relief and dynamic analysis at the first execution of step 3 could be used to check the error of inertia relief.

Step 4. Update the design variables and the global set point based on HCA update rule, obtain an improved design and check its convergence. If the convergence criterion is reached go to step 5. If not, go back to step 3.

Step 5. Carry out transient dynamic analysis of the improved design with the same initial conditions and find the new peak load. If the change of the peak load is small enough, the optimization process is finished.

5 Numerical example of new hybrid method

5.1 A simplified car model

A complex car model is simplified to two parts, one is the car body with concentrated masses, and the other is the bumper part to protect the car, shown in figure 4. The total mass of the concentrated masses at the front are 600Kg, and the mass of the masses in the rear are 300Kg. There are four masses placed on the location of the passengers in the car body, each of them is 100Kg. At last, four masses on the location of the tires, and each of them is 50 Kg. We use fully integrated selectively-reduced 8-node brick hexahedral element to divide design domain in order to use the HCA method. The material's parameters of the design area are, density is 785Kg/m^3, elastic modulus is 2.1e10Pa, yield stress is 2.1e5Pa, and tangent modulus is 1e9Pa. For the material's parameters of the bumper, material density is 785Kg/m^3, elastic modulus E is 2.1e9Pa, yield stress is 2.1e4Pa, and tangent modulus is 1e8Pa. The simplified car model contains 26961 elements, 34662 nodes, and the design domain contains 25076 elements. The tolerance of the peak load change ΔP_t is 5% of the peak load of previous peak load.

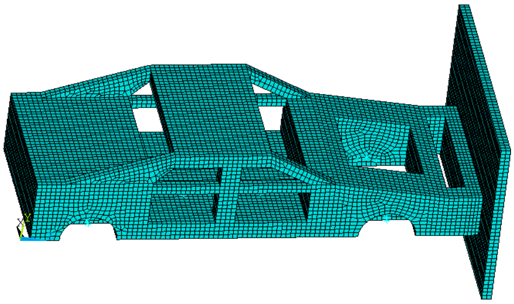
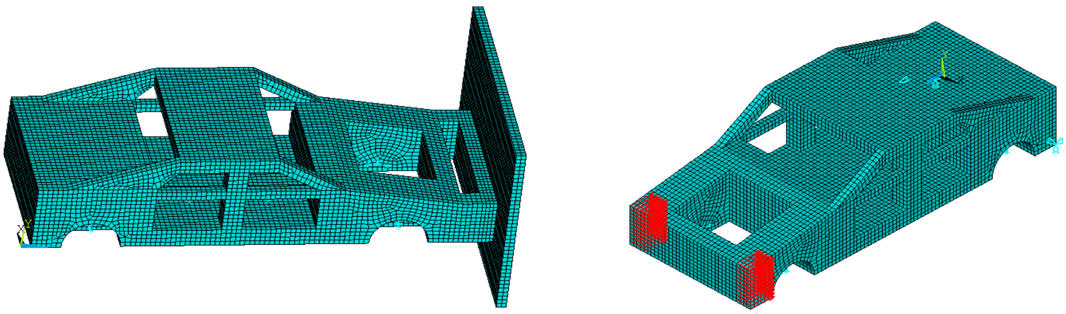


Figure 4. Car model of topology optimization

5.2 Optimization process of the new hybrid method

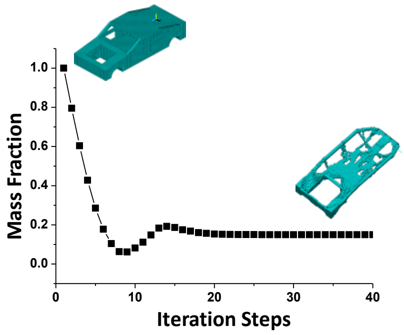
In this example, the impact load of the body part is transformed from the bumper part, which leads to uniform stress distributions of the impact load's areas. Thus, the static load on different nodes of inertia relief analysis model has same magnitude. The structural models for transient dynamic analysis and inertia relief analysis are shown in figure 5. The dynamic analysis uses Ls-dyna program, and the model is meshed by fully integrated selectively-reduced 8-node brick hexahedral element. The inertia relief analysis uses Ansys program, and the model is meshed by Full integration 8-node brick hexahedral element.



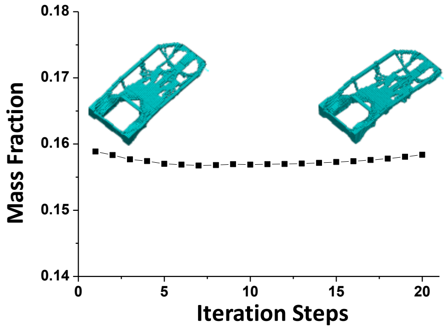
(a) The model of dynamic analysis **(b) The model of inertia relief**

Figure 5. The analysis models and stress distributions of two methods

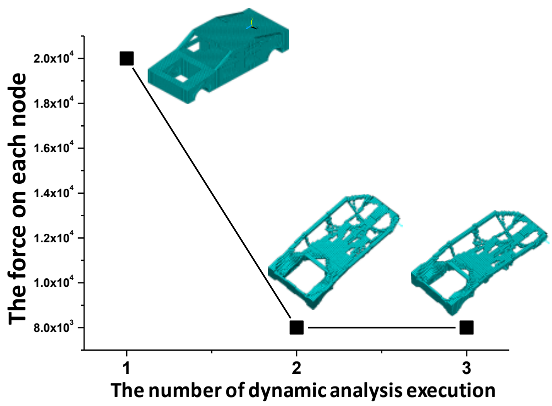
The iteration process of the new hybrid method for the mass fraction constraint M_f^*=0.16 is shown in figure 6. Three dynamic analyses totally execute and the inner loop executes twice. The first dynamic analysis is to find the peak impact load of the initial design, which is 2.0e4N on each node in the crash area. The second dynamic analysis changes the peak load of the optimized design, and the peak load of each node reduced to 0.8e4N. After the third dynamic analysis, the peak load is still 0.8e4N, and then the optimization is finished because of reaching the convergence condition. There are 40 iterations in first inner loop and 20 iterations in second inner loop. The inner loop is the optimization process of the HCA method. By our computer, the time cost of the dynamic analysis is about 20 minutes, but the inertia relief method just cost 2 or 3 minutes for this model. Though extra 20 iterations needed, the new hybrid method still reduces the time cost of the optimization significantly. In this problem, the structural nonlinearity is not significant, and the optimized design after first inner loop is almost the same as the optimized design after second inner loop. So for the similar problem, maybe we just need two inner loops to find the optimized design and the time cost could be further reduced.



(a) The iteration of first inner loop



(b) The iteration of second loop



(c) The iteration of outer loop

Fig. 6 The optimization process of the new hybrid method (mass fraction is 0.16)

Now let us compare the optimum design under the mass fraction constraint M_f^*=0.16 with the initial full design. However, the kinetic energy will be different for these two models because the initial full design is more heavy than the optimum design under the mass fraction constraint M_f^*=0.16. In order to compare the designs, we reduce the density of the full design to make the kinetic energy equal with the optimized design. The following comparison is based on the model of reduced density.

First we compare the energy absorption ability. The energy absorption history curves are shown in figure 7, the curve of the optimized design is the red one, the maximum energy absorption is 16.58KJ, and the blue one is for the initial design, the maximum energy absorption is 11.44KJ. The energy absorption ability of the optimized design is enhanced about 45% than the initial design.

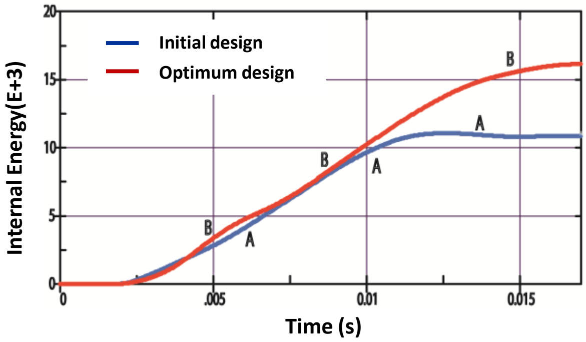


Figure 7. Energy absorption-time curve of initial and optimized design

Conclusions

The presented new hybrid method takes advantage of HCA and inertia relief method and provides an alternative approach to structural topology optimization of vehicle for crashworthiness design. However, it needs further improvement, more elaborated investigation and more complicated test examples.

In this paper, we propose a new hybrid method to vehicle energy absorption optimization problem. Then we construct a simplified car body model and test the hybrid method. The results show that the hybrid method could reduce the computational cost of the optimization significantly and get a better design of the crashworthiness concept design. And the proposed approach converges very fast.

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