Simulation of Nonlinear Magnetorheological Particle-filled Elastomers

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Polymer matrix filled with ferromagnetic particles is a class of smart materials whose mechanical properties can be changed under different magnetic field. They are usually referred to as magnetorheological elastomers (MREs). A finite element simulation was presented to describe the mechanical behavior of MREs with the nonlinearity of the particle magnetization being incorporated. By introducing the Maxwell stress tensor, a representative volume element (RVE) was proposed to calculate the Young's modulus and shear modulus of MREs due to the applied magnetic field. The influences of the applied magnetic field and the particle volume fractions in the shear modulus and Young's modulus were studied. Results show that the shear modulus increases with the magnitude of the applied magnetic field, while the Young's modulus decreases.

Keywords: Magnetorheological elastomers; Mechanical properties; Maxwell stress tensor; Representative volume element

Introduction

Magnetorheological elastomers (MREs) are a new material, whose structure is that micro-sized ferrous particles disperse in a polymer matrix. The materials have rheological properties that can be changed by an applied magnetic field continuously, rapidly and reversibly. The increasing interest in MREs has heightened the need for potential applications in vales, dumpers,brakes and sensor constructions. Some theoretical models were established to simulate MREs. The widely used one now is the dipole model (Davis,1999; Jolly,1996; Shiga and Okada,1995). However, it puts the dipole as the center of the sphere thus being limited to the case of larger particle spacing. The dipole theory indicates that the ferromagnetic particles are linear magnetization. In fact, the particles are nonlinear. The distribution of magnetic field in MREs can be calculated by the magnetic finite element method. The shear modulus, the particle volume fraction and the relationship between the size of the magnetic field were obtained by considering the nonlinear magnetization of particles. This work can provide guidance for the optimized material and device design.

Modeling procedure

Representative volume element

As shown in Fig. 1, a representative volume element was used to simulate MREs. Two dimension square RVE was regarded as the rubber matrix, which includes several circles representing carbony1 iron particles CIPs.

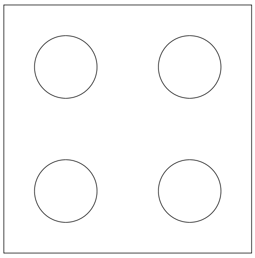

Fig. 1 RVE model

Governing equations

Cauchy's equation of continuum mechanics reads

$$\rho \frac{d^2\boldsymbol{r}}{dt^2} = \nabla \cdot \boldsymbol{T} + \boldsymbol{f}_{ext} \tag{1}$$

where ρ is the density, $\boldsymbol{r}$ is the coordinates of a material point, $\boldsymbol{T}$ is the stress tensor, and $\boldsymbol{f}_{ext}$ is an external volume force such as gravity ($\boldsymbol{f}_{ext}=\rho\boldsymbol{g}$). It is an equation solved in the structural mechanics physics interfaces in the special case of a linear elastic material, which neglects the electromagnetic contributions. In the stationary case, there is no acceleration, and the equation representing the force balance is

$$\boldsymbol{0} = \nabla \cdot \boldsymbol{T} + \boldsymbol{f}_{ext} \tag{2}$$

In certain cases, the stress tensor $\boldsymbol{T}$ can be divided into two parts. One depends on the electromagnetic field quantities and another is the mechanical stress tensor,

$$\boldsymbol{T} = \boldsymbol{T}_{EM} + \boldsymbol{\sigma}_M \tag{3}$$

It is sometimes convenient to use a volume force instead of the stress tensor. This force is obtained from the relation

$$\boldsymbol{f}_{em} = \nabla \cdot \boldsymbol{T}_{EM} \tag{4}$$

as stated in the structural mechanics physics interfaces

$$\boldsymbol{0} = \nabla \cdot \boldsymbol{\sigma}_M + \boldsymbol{f}_{em} + \boldsymbol{f}_{ext} \tag{5}$$

Magnetic equations

In a current free region, where $\nabla \times \boldsymbol{H} = 0$. It is possible to define the scalar magnetic potential, V_m from the relation $\boldsymbol{H} = \nabla V_m$. Using the constitutive relation between the magnetic flux density and magnetic field

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \tag{6}$$

together with the equation

$$\nabla \cdot \boldsymbol{B} = 0 \tag{7}$$

then V_m can be obtained from the equation

$$-\nabla\cdot(\mu_0\nabla V_m-\mu_0\boldsymbol{M}_0)=0 \tag{8}$$

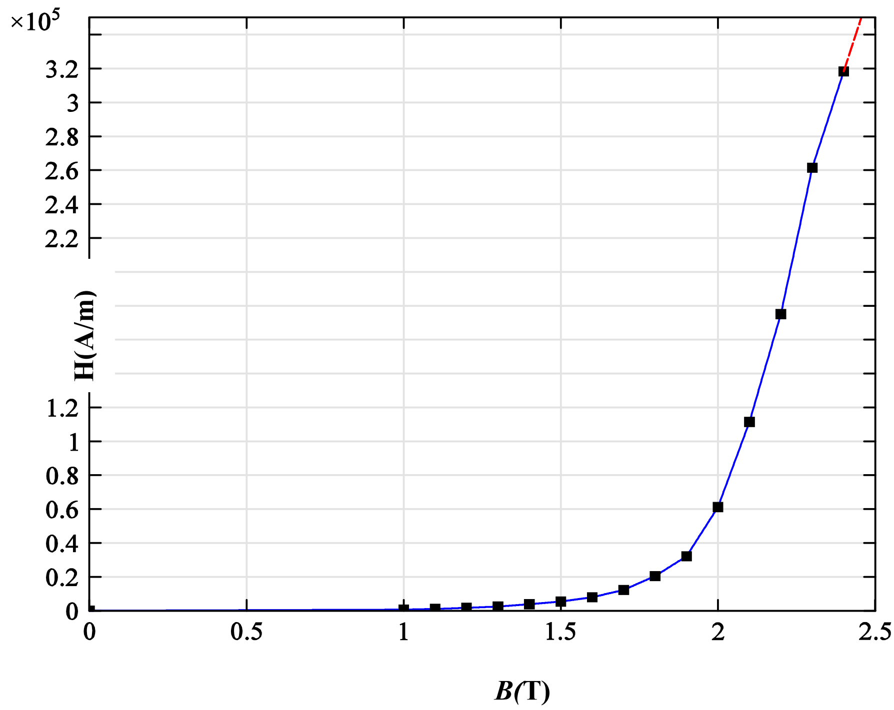

Fig. 2 HB curve

The nonlinear magnetic behavior of the steel particles is molded by using a HB curve to specify the magnetic constitutive relation.（Fig. 2）

Mechanical equations

The expressions for the stress tensor in a general electromagnetic context stem from a fusion of material theory, thermodynamics, continuum mechanics, and electromagnetic field theory. With the introduction of thermodynamic potentials of mechanical, thermal, and electromagnetic effects, explicit expressions for the stress tensor can be derived in a convenient way by forming the formal derivatives with respect to the different physical fields (Kovertz,1990;Wilson,1988). Alternative derivations can be made for a vacuum (Wangsness,1986) but it is difficult to polarize and magnetize materials. In general, an elastic solid material of that is dielectric and magnetic (nonzero M), the stress tensor is given by the expression

$$\boldsymbol{T}=\boldsymbol{\sigma}(\boldsymbol{B})+\frac{1}{\mu_0}\boldsymbol{B}\cdot\boldsymbol{B}^T-\boldsymbol{M}\boldsymbol{B}^T-\frac{1}{2\mu_0}\left(\boldsymbol{B}\cdot\boldsymbol{B}-\boldsymbol{M}\cdot\boldsymbol{B}\right)\mathbf{I} \tag{9}$$

where in $\boldsymbol{\sigma}(\boldsymbol{B})$, the dependence of $\boldsymbol{B}$ has not been separated out. Thus $\boldsymbol{\sigma}$ is not a purely mechanical stress tensor in this case. Different material models give different appearances of $\boldsymbol{\sigma}(\boldsymbol{B})$. The electromagnetic contributions to $\boldsymbol{\sigma}(\boldsymbol{B})$, which represent piezoelectric, dielectric, and magnetization effects. The expression for the stress tensor in vacuum, air, and pure conductors can be derived from this general expression by setting $\boldsymbol{M}=0$. The Maxwell stress on CIPs causes the deformation of the RVE.

Simulations procedures

A test of the MREs was simulated to investigate the changes of shear modulus and Young's modulus with different magnetic field and radius of CIPs. The values of magnetic field are 0.5e-5wb/m, 1e-5wb/m, 1.5e-5wb/m, and 2e-5wb/m (Fig.3 and Fig.4), respectively. The radius of CIPs is 1.5 μm， 2 μm， 2.5 μm， 3 μm， 3.5 μm， 4 μm， 4.5 μm (Fig.5), respectively.

Results and discussions

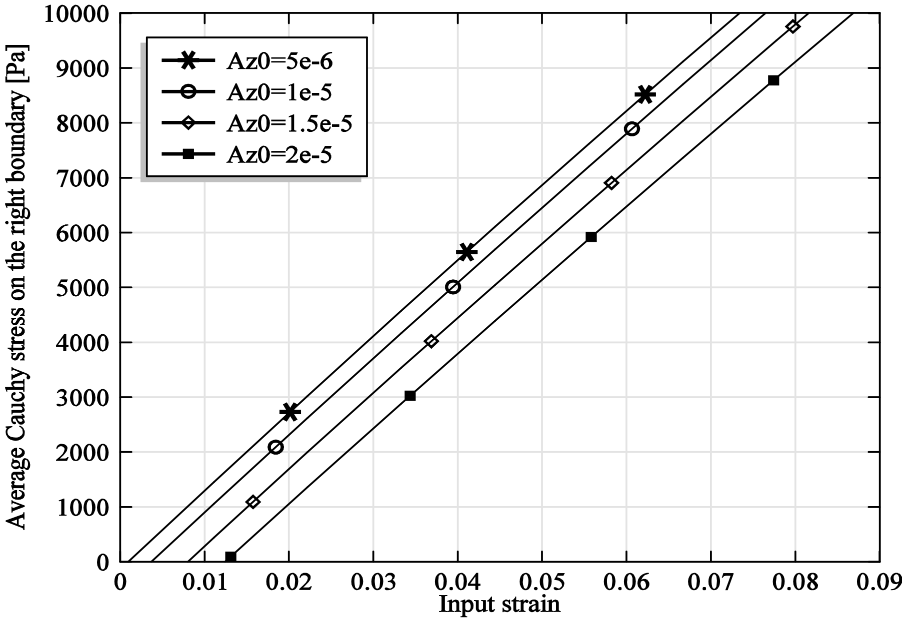

Fig. 3 different magnetic fields for tension deformation

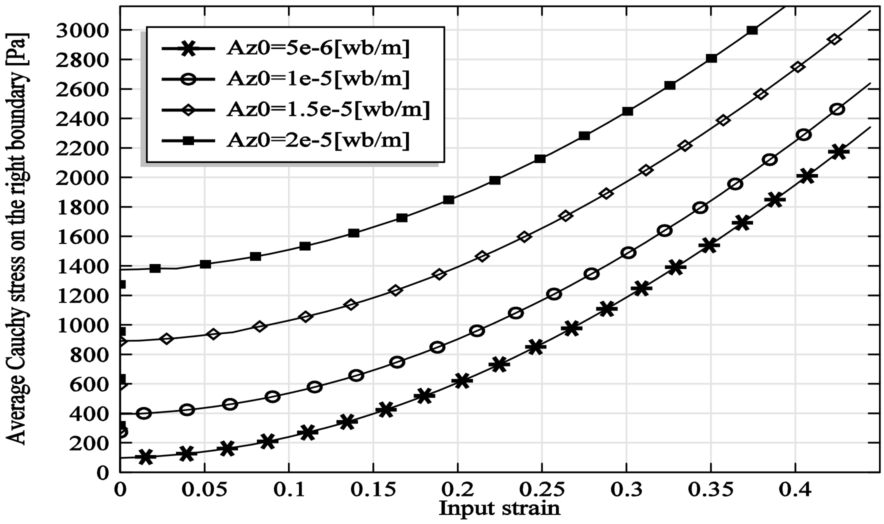

Fig. 4 different magnetic fields for shear deformation

When no shear deformation occurs, $\gamma = 0$, the magnetic field contribution to the normal stress and shear strain is equal to the magnetic energy density function of the first derivative of strain,

$$\sigma_B = \frac{\varphi M^2 d_p^3 \mu_0}{4r^3} - \frac{\varphi M^2 d_p^3 \mu_0}{r_0^3}\varepsilon \tag{10}$$

Young's modulus is:

$$\Delta E(B) = \frac{\varphi M^2 d_p^3 \mu_0}{r_0^3} \tag{11}$$

As stated in the above equation, the magnetic field will cause magnetic force and the load will result in compression of the matrix (rubber), which is actually magnetic pre-stressed. This is because that the CIPs attract each other, which increases the reliability of material in some ways. But in the contact force between the particles and the rubber, the rubber is easy to be torn. The MREs working in the compressed state are unreasonable (the compressed state becomes the extrusion of the two rigid iron balls). Young's modulus induced by the magnetic field is negative. Its physical meaning is the magnetic energy and a matrix strain energy conversion. With increasing the magnetic field (Fig.3). MREs will become softer. The results of the simulation show that the shear modulus increases with the magnitude of the applied magnetic field, while the Young's modulus decreases (Fig.4).

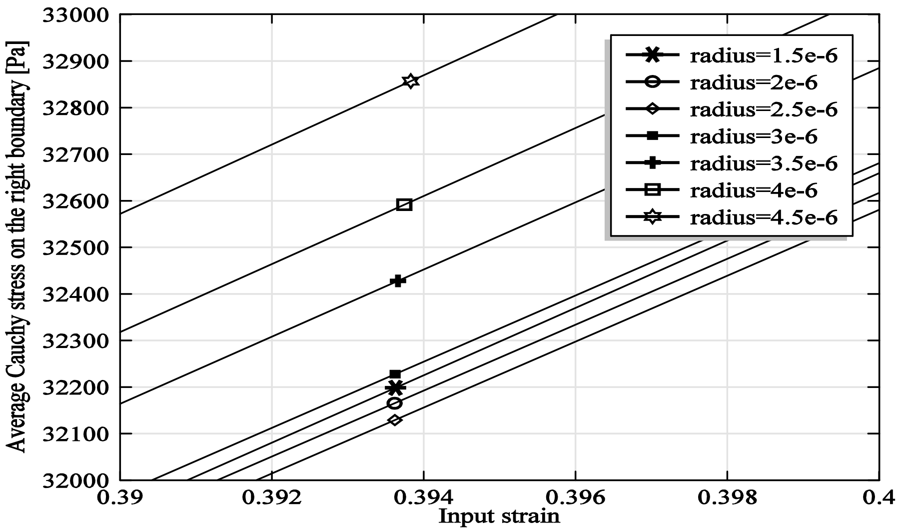

Fig. 5 different radius of CIPs

Conclusions

The study focuses on the mechanical behavior of MREs. A two-dimension RVE was introduced with CIPs and pure rubber. Mechanical behavior of the RVE was simulated under magnetic field. It is demonstrated that the shear modulus increases with the magnitude of the applied magnetic field, while the Young's modulus decreases. Additional research should focus not only upon the applied magnetic field, but also considering the magnetostriction of MREs.

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