Uncertainty in long-term behavior and buckling of concrete-filled steel tubular columns

*** Xue Shi, Wei Gao, Yong-Lin Pi, Mark A. Bradford [1]**

[1]School of civil and environment engineering, University of New South Wales, Australia.

Abstract

This paper presents a long-term and buckling analysis of concrete-filled steel tubular (CFST) columns under sustained axial compression by accounting for the uncertainties of creep and shrinkage of the concrete core. The intervals of the final shrinkage strain and final creep coefficient of concrete core are derived from test results. An interval analytical model based on the algebraically tractable age-adjusted effective modulus method is proposed for the uncertain long-term and buckling analysis of CFST columns. An interval finite element model was developed in this paper for long-term behavior and buckling analysis. Perturbation method was employed to determine the two bounds of the solution. The results of the proposed analytical model and finite element model were compared with experimental results and analyzed.

Keywords: creep, shrinkage, interval analysis, interval finite element analysis, perturbation method

1. Introduction

Concrete-filled steel tubular (CFST) columns have been used in construction since the mid-1980s (Schneider, 1998) and become increasingly popular in both high-rise buildings and bridges (Shams and Saadeghvaziri, 1997). A CFST section consists of a steel tube and a concrete core (Fig. 1). Creep and shrinkage of the concrete core occur with an increase of time, which influence the long-term behavior of CFST columns significantly. It is of great importance to correctly predict effects of creep and shrinkage of the concrete core on the long-term behavior of CFST columns.

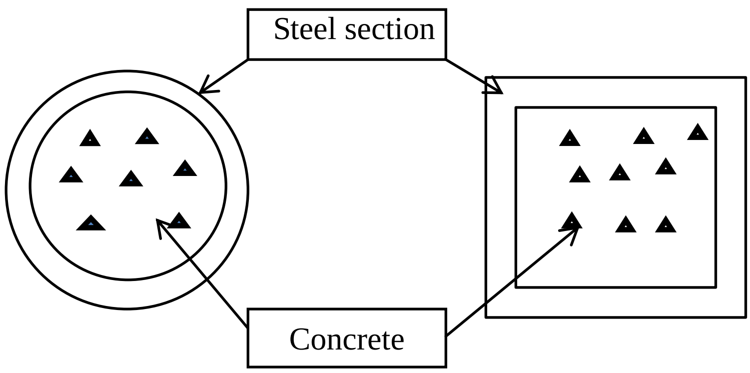

Figure 1. Cross-sections of CFST columns

Experimental and analytical studies have been performed by pioneers over the past three decades. Terrey et al. (1994) conducted similar experiments on circular CFST columns axially loaded at an earlier age of the concrete core. The first test on square CFST columns was carried out by Morino et al. (1996)and six concentrically loaded columns, two eccentrically loaded columns and one flexural member were tested. Experiments for similar cross-sections were implemented by Uy (2001) applying axial loading to the CFST columns at 14 and 28 days of age of the concrete core respectively. Square CFST columns subjected to higher sustained loads were investigated by Han et al. (2004). Terry et al. (1994), Uy (2001) and Han et al. (2001) predicted the concrete time-dependent behavior by using the age-adjusted effective modulus (AAEM) method proposed by ACI-209. Cheng et al. (2005) introduced a three-dimensional nonlinear laminated element into the long-term modeling and assumed the creep behavior to be described by the Kelvin model. All of these experimental and analytical investigations treated the creep and shrinkage behaviors of the concrete core as deterministic phenomena.

However, it is noted that the creep coefficient obtained from tests vary significantly from one experiment to another. Very different predictions of the time-dependent behavior of CFST columns have been reported in different studies. This shows that the uncertainties of creep and shrinkage of the concrete core do exist. To predict the long-term behavior of CFST columns reasonably, these uncertainties have to be considered. Uncertain analysis of engineering structures has been developed in the last two decades and probabilistic methods are usually used if sufficient probabilistic information is available to validate the distributions or probability density functions of random variables. Other non-probabilistic approaches such as interval arithmetic and fuzzy sets theory are excellent alternatives when the statistical data of variables are not enough. For the long-term analysis of CFST columns considering the creep and shrinkage of the concrete core, probabilistic methods require probabilistic distributions of the final creep coefficient and final shrinkage strain to be determined first. Unfortunately, the available test data for creep and shrinkage of the concrete core of CFST sections are quite limited. Hence, it is impossible to derive correct probabilistic distributions of the final creep coefficient and final shrinkage strain.

In this paper, intervals are adopted to represent the uncertainties. In interval model, only the lower and upper bounds are required, which will be determined from the currently available experiment results for the final creep coefficient and final shrinkage strain. The age-adjusted effective modulus method (AEMM) (Bažant, 1972) is used to describe the creep of the concrete core and a virtual work method is used to establish the differential equation for the long-term-analysis of CFST columns that are subjected to a sustained axial uniform compression. Interval analyses are then implemented to predict the uncertain long-term behavior of CFST columns and buckling loads caused by the variations of the creep and shrinkage of the concrete core. Finally, an extensive parametric study is carried out to evaluate the influence of time, load level, steel ratio for CFST columns.

2. Interval analytical analysis

2.1 Interval linear elastic analysis of long-term behavior of CFST column

To predict the long-term performance, interval constitutive model considering creep and shrinkage of the CFST column needs to be established. The basic assumptions adopted for the interval long-term linear elastic analysis of CFST columns in this paper are: (1) deformations of the CFST columns are elastic and satisfy the Euler–Bernoulli hypothesis; and (2) the steel tube is fully bonded with the concrete core. Then, the linear strain of the CFST column can be expressed as

$$\varepsilon = u' \tag{1}$$

where u is the vertical displacement along the longitudinal direction of the CFST column. Based on the age-adjusted effective modulus method (AEMM), the stress in concrete can be expressed as

$$\sigma_c = E_{ec}(\varepsilon + \varepsilon_{sh}) = E_{ec}(u' + \varepsilon_{sh}) \tag{2}$$

where E_{ec} is the age-adjusted effective modulus of concrete, ε_{sh} is the shrinkage strain of concrete and can be given by AS3600 (2001)

$$\varepsilon_{sh}\left(t\right)=\frac{\varepsilon_{shfinal}}{35+t}\cdot t \tag{3}$$

where t is the loading time, $\varepsilon_{shfinal}$ is the final shrinkage strain of concrete when $t \rightarrow\infty$. E_{ec} can be calculated by

$$E_{ec}(t,\tau_0)=\frac{E_c}{1+\chi(t,\tau_0)\varphi(t,\tau_0)} \tag{4}$$

where τ_0 is the age at loading, $\chi(t,\tau_0)$ is the aging coefficient and $\varphi(t,\tau_0)$ is the creep coefficient that can be expressed as

$$\varphi(t,\tau_0)=[\frac{(t-\tau_0)^{0.6}}{10+(t-\tau_0)^{0.6}}]\cdot\varphi_{final} \tag{5}$$

where φ_{final} is the final creep coefficient when $t \rightarrow\infty$. The aging coefficient $\chi(t,\tau_0)$ can be expressed as (Gilbert, 1988)

$$\chi(t,\tau_0)=1-\frac{\left(1-\chi^*\right)\left(t-\tau_0\right)}{20+\left(t-\tau_0\right)} \tag{6}$$

where

$$\chi^*=\frac{k_1\tau_0}{k_2+\tau_0} \tag{7}$$

with

$$k_1=0.78+0.4e^{-1.33\varphi_{\infty,7}} \tag{8}$$

$$k_2=0.16+0.8e^{-1.33\varphi_{\infty,7}} \tag{9}$$

$$\varphi_{\infty,7}=\varphi_{final}t_0^{0.118}/1.25 \tag{10}$$

As the concrete core is assumed to be fully bonded with the steel tube, the deformations of the steel and concrete must be compatible with each other. Consequently, their mechanical membrane strains are equal to each other and the mechanical strains at the interface between the steel tube and concrete core are the same. Therefore, the stress σ_s in the steel tube can be written as

$$\sigma_s=E_s\varepsilon=E_s u^{'} \tag{11}$$

where E_s is the Young's modulus of steel.

The differential equations for the long-term analysis of the CFST column can be obtained using the virtual work method. When the virtual work principle is used for the long-term equilibrium of the CFST column, it can be stated as

$$\delta W=\int_{V_s}\sigma_s\delta\varepsilon dV+\int_{V_c}\sigma_c\delta\varepsilon dV-Pu=0 \tag{12}$$

where V_s is the volume of the steel tube, V_c is the volume of the concrete core, δ ( ) denotes the Lagrange operator of simultaneous variations. By substituting Eqs. (1), (2) and (4), the statement of the principle of virtual work given by Eq. (13) can be written as

$$\delta W=\int_0^L(A_sE_s+A_cE_{ec})u^{''}\delta udy+(A_s\sigma_s+A_c\sigma_c-P)\delta u\Big|_0^L=0 \tag{13}$$

Integrating Eq. (13) by parts leads to the differential equation of equilibrium for the long-term behavior of CFST column

$$u^{''}=0 \tag{14}$$

and leads to the static boundary condition for CFST column as

$$A_s\sigma_s + A_c\sigma_c - P = 0 \text{ at } x=L \tag{15}$$

where L is the length of the CFST column. The essential geometric boundary condition is

$$u = 0 \text{ at } x = 0 \tag{16}$$

The long-term displacement of the CFST column can be obtained from Eq. (14) - (16) as

$$u = \frac{P + A_c E_{ec}\varepsilon_{sh}}{A_s E_s + A_c E_{ec}} x \tag{17}$$

and the strain of the steel tube and concrete core can be obtained from Eq. (17) as

$$\varepsilon = u' = \frac{P + A_c E_c \varepsilon_{sh} / [1+\chi(t,\tau_0)\varphi(t,\tau_0)]}{A_s E_s + A_c E_c / [1+\chi(t,\tau_0)\varphi(t,\tau_0)]} \tag{18}$$

In this paper, the final creep coefficient φ_{final} and the final shrinkage strain $\varepsilon_{shfinal}$ can be described in terms of interval variables as

$$\varphi^I{}_{final} = [\underline{\varphi_{final}}, \overline{\varphi_{final}}] \tag{19}$$

$$\varepsilon^I{}_{shfinal} = [\underline{\varepsilon_{shfinal}}, \overline{\varepsilon_{shfinal}}] \tag{20}$$

Based on the interval arithmetic and the deterministic solutions of long-term displacement, the interval long-term displacement of CFST column can be obtained from Eq. (17) as

$$\underline{u} = \frac{P + A_c E_c \underline{\varepsilon_{sh}} / [1+\overline{\chi}(t,\tau_0)\cdot\overline{\varphi}(t,\tau_0)]}{A_s E_s + A_c E_c / [1+\underline{\chi}(t,\tau_0)\cdot\underline{\varphi}(t,\tau_0)]} x \tag{21}$$

$$\overline{u} = \frac{P + A_c E_c \overline{\varepsilon_{sh}} / [1+\underline{\chi}(t,\tau_0)\cdot\underline{\varphi}(t,\tau_0)]}{A_s E_s + A_c E_c / [1+\overline{\chi}(t,\tau_0)\cdot\overline{\varphi}(t,\tau_0)]} x \tag{22}$$

and the interval strain of the steel tube and concrete core can be obtained from Eq. (18) as

$$\underline{\varepsilon} = \frac{P + A_c E_c \underline{\varepsilon_{sh}} / [1+\overline{\chi}(t,\tau_0)\cdot\overline{\varphi}(t,\tau_0)]}{A_s E_s + A_c E_c / [1+\underline{\chi}(t,\tau_0)\cdot\underline{\varphi}(t,\tau_0)]} \tag{23}$$

$$\overline{\varepsilon} = \frac{P + A_c E_c \overline{\varepsilon_{sh}} / [1+\underline{\chi}(t,\tau_0)\cdot\underline{\varphi}(t,\tau_0)]}{A_s E_s + A_c E_c / [1+\overline{\chi}(t,\tau_0)\cdot\overline{\varphi}(t,\tau_0)]} \tag{24}$$

2.2 *Interval buckling analysis*

The classic equilibrium equation for column can be expressed as

$$EI\frac{d^4 v}{dx^4} + P\frac{d^2 v}{dx^2} = 0 \tag{25}$$

where x denotes the axial coordinate, v is the transverse deflection, P is the applied axial force, E is the Young's modulus and I *is the* second moment of area. By using the kinematic boundary conditions that $v=0$ at $x = 0, L$ and the static boundary conditions, the solution of Eq. (25) can be obtained as

$$v = C_1 \sin \mu x \tag{26}$$

where μ is a time-dependent dimensionless axial force parameter defined by

$$\mu^2 = \frac{P}{E_s I_s + E_{ec} I_c} \tag{27}$$

where P can be express as

$$P = A_s E_s \varepsilon + A_c E_{ec} \varepsilon - A_c E_{ec} \varepsilon_{sh} \tag{28}$$

Considering the E_{ec} is an interval variable, Eq. (26) becomes

$$v = C_1 \sin \sqrt{\frac{A_s E_s \varepsilon_s + A_c E_{ec}{}^I \varepsilon_c - A_c E_{ec} \varepsilon_{sh}{}^I}{E_s I_s + E_{ec}{}^I I_c}} x \tag{29}$$

and the critical load can be obtain as when $\sqrt{\frac{A_s E_s \varepsilon_s + A_c E_{ec}{}^I \varepsilon_c - A_c E_{ec} \varepsilon_{sh}{}^I}{E_s I_s + E_{ec}{}^I I_c}} = n\pi$.

The interval critical load P_{cr}^I can be expressed as

$$P_{cr}^I = \frac{n^2 \pi^2 (E_s I_s + E_{ec}{}^I I_c)}{L^2} \tag{30}$$

when

$$\varepsilon^I = [\frac{n^2 \pi^2 (E_s I_s + \overline{E_{ec}} I_c) + A_c \overline{E_{ec}}\, \overline{\varepsilon_{sh}} L^2}{L^2 (A_s E_s + A_c \underline{E_{ec}})}, \frac{n^2 \pi^2 (E_s I_s + \underline{E_{ec}} I_c) + A_c \underline{E_{ec}}\, \underline{\varepsilon_{sh}} L^2}{L^2 (A_s E_s + A_c \underline{E_{ec}})}] \tag{31}$$

3. Interval finite element analysis

The equilibrium equations of the CFST column can be derived from the principle of virtual work that requires

$$dU = \int_{V_s} \{d\varepsilon^I\}^T \{\sigma_s{}^I\} dV + \int_{V_c} \{d\varepsilon^I\}^T \{\sigma_c{}^I\} dV - \{du^I\}^T \{p\} = 0 \tag{32}$$

The relationship between the stress and strain of concrete core can be expressed as

$$\{\sigma_c{}^I\} = [D_{ec}{}^I]\{\varepsilon^I + \varepsilon_{sh}{}^I\} \tag{33}$$

where $[D_{ec}{}^I]$ is the interval stress-strain matrix for concrete core. Similarly, the relationship between the stress and strain of the steel tube is

$$\{\sigma_s{}^I\} = [D_s{}^I]\{\varepsilon^I\} \tag{34}$$

Strains are determined from displacements, that is

$$\{\varepsilon^I\} = [B]\{u^I\} \tag{35}$$

where $[B]$ is the strain-displacement matrix.

Substituting Eqs. (33) to (35) into Eq. (32) yields

$$dU = \{du^I\}^T \{\int_{V_s} [B]^T [D_s^I][B]\{u^I\} dV + \int_{V_c} [B]^T [D_{ec}{}^I][B]\{u^I\}(1 + \frac{\varepsilon_{sh}{}^I}{\varepsilon^I}) dV - \{P\}\} = 0 \tag{36}$$

The CFST member is assumed to deform from the previous equilibrium state defined by {P} and {u} to an incremental equilibrium state defined by $\{P + \Delta P\}$ and $\{u + \Delta u\}$. Applying principle of virtual work, we can obtain

$$dU(u^I + \Delta u^I) = 0 \tag{37}$$

By using Taylor's series expansion, Eq. (40) becomes

$$\{\frac{\partial (dU)}{\partial u^I}\}^T \{\Delta u^I\} + \{\frac{\partial (dU)}{\partial p}\}^T \{\Delta p\} = 0 \tag{38}$$

Substituting Eq. (36) to Eq. (38), we have

$$\{du^I\}^T K(u)_T{}^I \Delta u^I = \{du^I\}^T \Delta P(u^I) \tag{39}$$

where Δu^I is the increment of interval displacement of the structure, $\Delta P(u^I)$ is the increment of load. $K(u)_T{}^I$ is the interval tangent stiffness matrix of the structure and can be expressed as

$$K_T{}^I(u) = \int_{V_s} [B]^T [D_s^I][B] dV + \int_{V_c} [B]^T [D_{ec}{}^I][B] dV + \int_{V_c} [B]^T (\frac{\varepsilon_{sh}{}^I}{\varepsilon^I}[D_{ec}{}^I])[B] dV$$

$$= K_s^I + K_{ec}{}^I + K_{sh}{}^I \tag{40}$$

of which K_s^I is the linear interval elastic stiffness matrix for steel tubular, $K_{ec}{}^I$ is the interval effective stiffness matrix for concrete core, and $K_{sh}{}^I$ is the interval strain stiffness caused by shrinkage. $K_{ec}{}^I$ and $K_{sh}{}^I$ are both dependent on time. The tangent stiffness matrix is updated after each increment of load or each increment of displacement due to creep and shrinkage.

4. Model validation and discussions

4.1 Determination of intervals for the final shrinkage strain and creep coefficient

The empirical values of the final shrinkage strain $\varepsilon^I{}_{shfinal}$ and creep coefficient $\varphi^I{}_{final}$ are proposed in several experimental studies. The value of the final shrinkage strain given by Han et al. (2004), Morino et al. (1996), Terrey et al. (1994) and Uy (2001) is 43.5, 83.6, 50 and 160, respectively. Correspondingly, the final creep coefficient is 0.5, 0.83, 1.0 and 1.5 respectively. These values vary considerably. To account for these variations in the long-term analysis of CFST columns, the interval of the final shrinkage strain and creep coefficient of their concrete cores can be derived from these test results as $\varepsilon_{shfinal}$ = [43.5, 340] and φ_{final} = [0.5, 1.7] respectively, which are used in this study. It can be expected that the results obtained by the interval models proposed in this paper will contain these experimental results, in other words, the experimental results will fall into the interval bounds produced by the proposed models.

4.2 Long-term behavior of CFST column by interval analytical analysis

Han et al. (2004) carried out long-term tests on CFST square section columns. The dimensions of the square section are 100 mm and the thickness of the square section is 2.93 mm. The length of the CFST columns is L = 600mm. Young's modulus of the steel tube E_s = 202 $\times 10^3$ MPa and Young's modulus of the concrete core E_c = 29200 MPa. The first loading time is 28 days after concrete core casting. A central axial load of 360 kN was applied to the CFST columns.

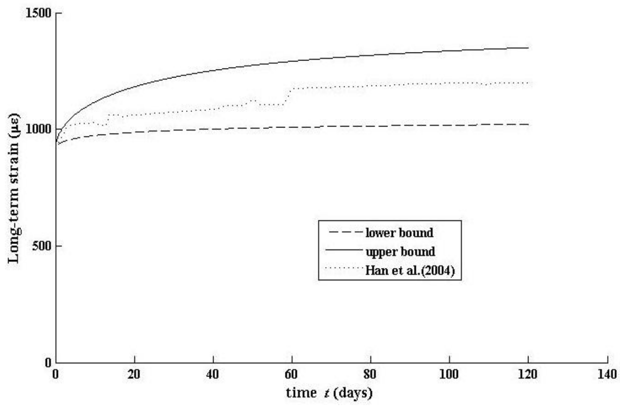

Figure 2. Comparison of creep and shrinkage strains

The analytical interval solution for the strain of CFST columns is compared with the test results in Figure 2. It can be observed that the interval uncertainty analysis can provide good upper and lower bounds for test results.

4.3 Interval numerical buckling analysis

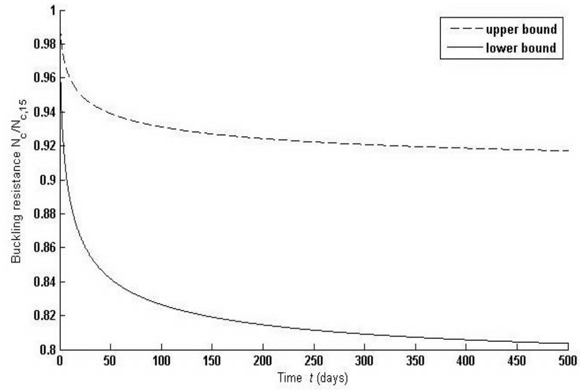

Figure 3. Interval critical buckling load on long-term sustained loading

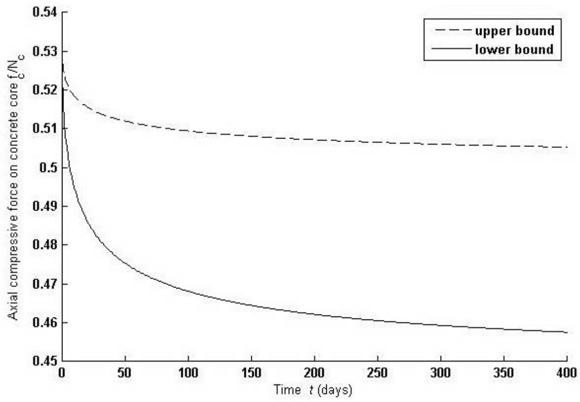

Figure 4. Compression force on concrete core when buckling

Fig. 3 shows the interval critical buckling load under long-term sustained load obtained by the interval finite element analysis method. Figs.4 shows the compression force on concrete core f_c / N_{cr} with time t, respectively. Young's modules of the steel and concrete are $E_s = 200GPa$ and $E_c = 30GPa$ .The size of section is $100\text{mm} \times 100\text{mm} \times 3\text{mm}$. The length is 1000 mm.

It can be seen from Fig. 3 that when time *t* increases, the critical buckling load decreases significantly due to the effects of the creep and shrinkage. The decrease range is [7%, 20%] at t=300days. Fig. 4 shows that, along the time *t*, the "buckling resistance contribution" from the concrete is decreasing while form the steel tubular is increasing.

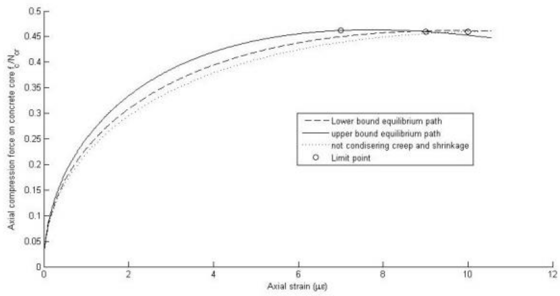

Figure 5. Axial strain for limit point buckling

Fig. 5 shows the compression force on the concrete core with axial strain. Buckling is investigated in a way that the column under an incremental load and the load criterion applies (Zhou, 2010). When creep buckling tends to happen, increment of the load becomes very small. From these figures, it can be seen that the axial strain is becoming smaller when buckling happens, in other words, the buckling resistance of the column is decreased due to the creep and shrinkage.

5. Conclusions

This paper presents a theoretical study on the uncertain long-term and buckling analysis of concrete-filled steel tubular columns. An interval analytical model based on the algebraically tractable age-adjusted effective modulus method is proposed to describe the time-dependent behavior of concrete in CFST columns. The solution of this model is compared with the experimental results reported by other researchers, which show the good agreements. Based on the energy method, the formulations for elastic buckling of the steel plate in rectangular CFT columns under axial compression are derived. An interval finite element was developed to describe the long-term behavior and analysis buckling. The buckling load or buckling time can be evaluated using this model.

In the future, the proposed models will be further developed to analyze other types of CFST structures accounting for the uncertainties in their material and geometric properties.

Reference

1. Schneider, S. P. (1998), ''Axially loaded concrete-filled steel tubes.'' *J. Struct. Eng.,* 124(10), pp.1125–1138
2. Shams M, Saadeghvaziri MA.(1997), State of the art of concrete-filled steel tubular columns. *ACI Struct J*; 94(5), pp.558–71.
3. Terrey, P. J., Bradford, M. A., & Gilbert, R. I. (1994), Creep and shrinkage of concrete in concrete-filled circular steel tubes. *Proc.,6th Int. Symposium on Tubular Structures*, Melbourne, Australia, pp.293–298.
4. Morino, S.; Kawaguchi, J.; & Cao, *Z.* S. (1996), Creep Behavior of Concrete-Filled Steel Tubular Members. *Composite Construction in Steel and Concrete 111*. Proceedings of an Engineering Foundation Conference, Irsee, Germany, pp.514-525.
5. Uy, B. (2001), Static long-term effects in short concrete-filled steel box columns under sustained loading. *ACI Structural. Journal* 98(1), pp. 96–104.
6. Han, L. H., Tao, Z. & Liu, W. (2004), Effects of Sustained Load on Concrete-Filled Hollow Structural Steel Columns. *J. Struct. Eng* 130(9), pp.1392–1404.
7. Cheng, X.D., Li, G.Y. & Ye, G.R. (2005), Three-dimensional nonlinear analysis of creep in concrete filled steel tube columns. *Journal of Zhejiang University (Science)* 6A(8), pp. 826–835.
8. Bažant, Z.P. (1972), Prediction of concrete creep effects using age-adjusted effective modulus method. *ACI Structural Journal* 69(4), pp.212–217.
9. AS3600,(2001), Australian Standard: Concrete Structures, Standard Association of Australia, Sydney.
10. R.I. Gilbert, (1988), Time Effects in Concrete Structures, Elsevier Applied Science, Amsterdam.
11. H.Z. Zhou. (2010), Buckling of reticulated laminated veneer lumber shells in consideration of the creep *Engineering Structures* 32, pp. 2912-2918