Numerical Simulations of Shock Wave Reflection over Double Wedges

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Abstract

Shock wave reflection is a fundamental topic in aerodynamic research area and has a wide application in engineering. Three-dimensional shock reflections over two perpendicularly intersecting wedges are numerically investigated in this paper by using the finite volume method with the MUSCL-Hancock interpolation technique and self-adaptive unstructured mesh. Mach stem structures are demonstrated to be three-dimensional (3D) ones and have special configurations at different wedge angles. There are two different kinds of 3D Mach stem structures for the MR–RR interaction, namely the first and the second types of 3D Mach stem, respectively. The three-shock or four-shock configuration may occur in the intersecting corner for the MR–MR interaction. The four-shock one is consisting of the incident shock wave, the 3D Mach stem, the primary and secondary reflected shock waves. In the RR–RR interaction, the incident shock wave, the primary and secondary reflected shock waves meet at the same reflection point to combine a three-shock configuration.

Keywords: Shock waves, Double Wedges, Reflection, Mach Stem

1 Introduction

Shock wave reflection is a fundamental topic in aerodynamic research and engineering applications. The phenomena of shock wave reflection were first investigated by Mach [1] in 1870s and the well-known 'Mach reflection (MR)' was later named after him. Different types of MR configurations were further demonstrated by von Neumann[2,3] in 1940s. In recent decades, shockwave reflections have been studied systematically, e.g., shock wave reflections over wedges the hysteresis phenomena in steady shock wave reflections[4–7], and the application of new experimental facilities[8]. However, the previous results are mainly on two-dimensional (2D) cases, and 3D shock wave reflection has not yet been investigated widely. This is mainly because the wave structures induced by 3D shock wave reflection are usually very complicated and thus difficult to be visualized clearly by the traditional visualization techniques.

Shock wave reflection over two perpendicularly intersecting wedges is schematically shown in Fig.1. This configuration of 3D shock reflection was first studied by Meguro et al.[9] experimentally, numerically and analytically. The 3D Mach stem was observed as well as its existence criterion according to the reflection types over each wedge, i.e., MR or regular reflection (RR). It was found that the 3D Mach stem definitely occurs for the MR–MR interaction, possibly occurs for the MR–RR interaction, but never occurs for the RR–RR interaction. The critical condition for whether or not the 3D Mach stem appears in the MR–RR interaction was analytically derived by the 2D theory of oblique shock wave reflection. In the case, as depicted in Fig.2, the 2D Mach stem on the vertical wedge was assumed to be two-dimensionally reflected over the horizontal wedge. Here, Mm□ denotes the Mach number of the Mach stem and θ_m is the angle between the intersecting line of the two wedges and the horizontal wall of the shock tube. θ_m corresponds to the inclination angle for the reflection of the Mach stem $M_{m'}$ over the horizontal wedge. $M_{m'}$ and θ_m can be calculated by geometry relations:

$$\theta_m = \arctan(\tan\alpha\cos\beta) \tag{1}$$

$$M_{m'} = M_s \cos\chi_\beta / \cos(\chi_\beta + \beta) \tag{2}$$

where χ_β is the triple point trajectory angle of the Mach reflection over the vertical wedge. If the assumed 2D reflection forms a Mach stem, namely A, then the Mach stems A and B would interact with each other and eventually result in a 3D Mach stem. The 3D shock wave reflections and the detailed interaction configurations are further investigated in this paper. Two kinds of 3D Mach stems and several types of shock wave reflection configurations are figured out. The existence criterion of the 3D Mach stem deduced from the 2D theory of shock wave reflection is re-examined using the computational results.

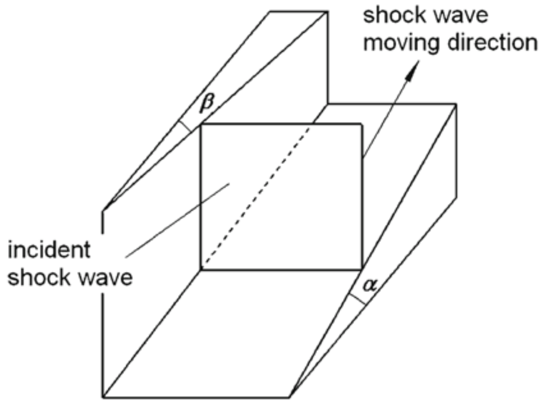


Fig.1 Shock reflection over two perpendicularly intersecting wedges

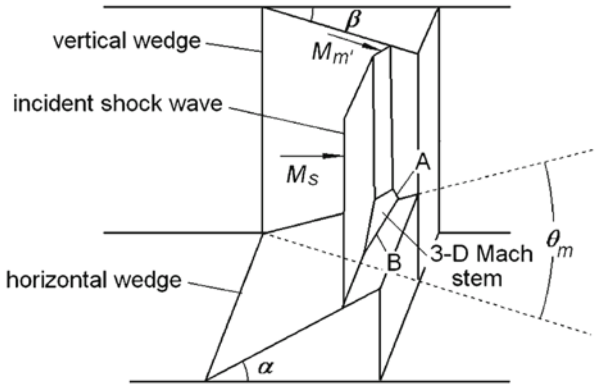


Fig.2 Regular-Mach reflection interaction

2 Governing equations and numerical methods

Assuming that viscosity effects on shock wave reflection are negligible, the governing equations are the hyperbolic system of three-dimensional conservation laws in Cartesian coordinates, which can be written as:

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}+\frac{\partial H}{\partial z}=0 \tag{3}$$

Where U, F, G and H denote the unknown variables and fluxes in the x-, y- and z-directions, respectively:

$$U=[\rho,\rho u,\rho v,\rho w,e]^T,\ F=[\rho u,\rho u^2+p,\rho uv,\rho uw,(e+p)u]^T,$$

$$G=[\rho v,\rho uv,\rho v^2+p,\rho vw,(e+p)v]^T,\ H=[\rho w,\rho uw,\rho uv,\rho w^2+p,(e+p)w]^T$$

The primitive variables in the unknown U are density ρ, velocity components u, v and w, and total energy per unit volume e. The equation of state for the perfect gas is given by:

$$e=\frac{p}{\gamma-1}+\frac{1}{2}\rho(u^2+v^2+w^2) \tag{3}$$

where p is the pressure and the specific heat ratio $\gamma = 1.4$.

Using the finite volume method, the equations can be converted to the integral form over the governing volume:

$$\int_{\Omega}\frac{\partial U}{\partial t}\mathrm{d}x\mathrm{d}y\mathrm{d}z+\int_{\Gamma}(F\mathrm{d}y\mathrm{d}z+G\mathrm{d}x\mathrm{d}z+H\mathrm{d}x\mathrm{d}y)=0 \tag{4}$$

where Ω and Γ denote the cubage and boundary of the governing volume, respectively. Hexahedron unit is adopted as the governing volume on the unstructured mesh and governing variables are fixed at the center of the unit. The HLLC scheme is applied to compute the fluxes on the governing unit boundaries and the second-order MUSCL scheme and the first-order time integral are used to reconstruct the governing variables at the unit center[10].

In the present numerical simulations, the boundary conditions on the wedge surfaces, the upstream boundary, the downstream boundary and the mainstream boundary are set to be the slipping solid condition, the inflow condition, the outflow condition and the mirror condition, respectively. The air ahead of the incident shock wave is motionless and the air behind is calculated by the Rankine–Hugoniot relations for a given shock wave Mach number.

3 Validation of numerical algorithms

The numerical algorithms are validated by comparing the numerical flowfield with the experiment results. The experiment is conducted in the 100mm × 180mm diaphragmless shock tube in the ShockWave Research Center, Tohoku University, Japan. The shock waves are visualized with double exposure diffuse holographic interferometry. In order to show the 3D shock waves clearly, the interval between the first and second exposure is set to be 1 μs.

Fig.3 shows the experimental and numerical results of the 3D shock wave reflection over two perpendicularly intersecting wedges. The incident shock wave Mach number is Ms = 2.0 and the inclination angles of the horizontal and vertical wedges are $\alpha = 43.5°$ and $\beta = 30°$, respectively. In Fig. 3a, it is obvious that the incident shock wave (I) is reflected over the wedges and a single-Mach reflection (R, M) appears over the horizontal wedge. Note that the reflected shock wave (R) over the vertical wedge is not as clear as the one over the horizontal wedge. It is mainly because the inclination angle of vertical wedge is relatively small and the reflected shock wave over it is relatively weak. However, it is still obvious that the reflection over the vertical wedge is a MR as the Mach stem (M) is visualized clearly. In the corner of the two intersecting wedges, the two Mach stems intersect each other forming a 3D forward-leaning Mach stem (M') followed by a secondary reflected shock wave (R'). Fig.3b shows the numerical result, which consists of three translucent isopycnic surfaces and the isopycnic lines in all the computational boundary planes. The isopycnic surfaces denote the shock waves in the 3D reflection. All the wave structures, such as the incident shock wave (I), the Mach stems (M,M), the reflected shock waves (R,R), the secondary reflected shock wave (R') and the 3D Mach stem (M'), can be identified clearly and agree well with the experimental result.

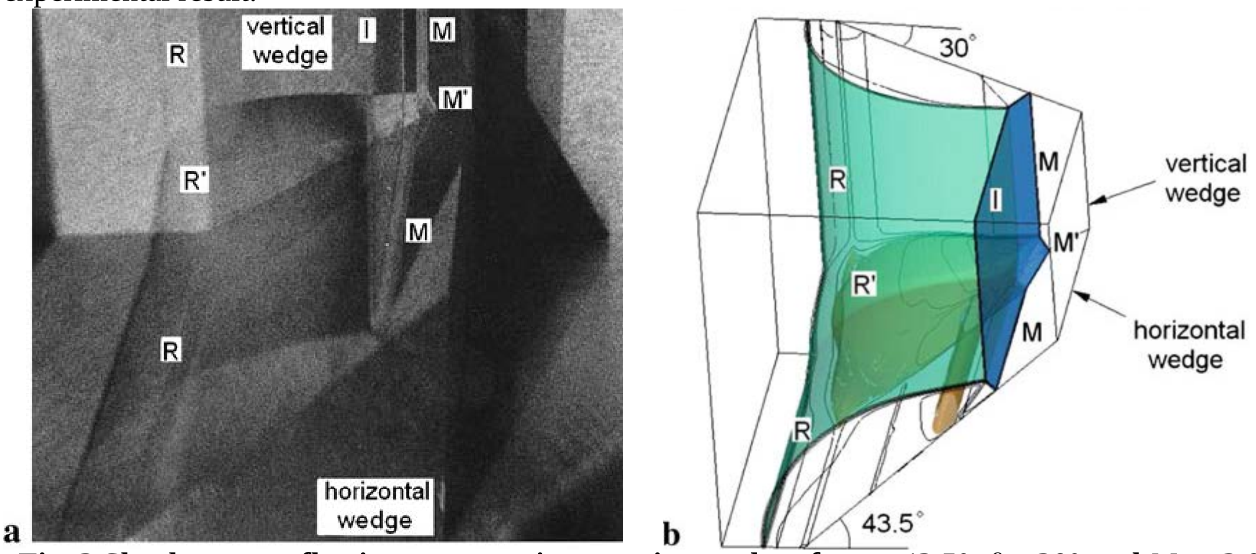


Fig. 3 Shock wave reflection over two intersecting wedges for α =43.5°, β **= 30° and** Ms = 2.0**: a) experimental result; and b) numerical result**

4 Results and discussion

4.1 MR–MR interaction

The shockwave reflection for the wedge angles $\alpha = \beta = 20°$ and incident shock wave Mach number Ms = 3.0 is shown in Fig. 4. Both of the patterns of the reflections over the vertical and horizontal wedges are single-Mach reflections. In the corner of the two intersecting wedges, the Mach stems intersect each other and a 3D Mach stem appears. It is obvious that the 3D Mach stem twists slightly and is approximately planar. Fig. 4 shows the 3D shock reflection from a different visual angle. In this figure the shockwave configuration in the corner of the two intersecting wedges can be observed clearly. The Mach stem on the vertical wedge is reflected over the horizontal wedge and a secondary MR appears. Similarly, the Mach stem on the horizontal wedge is also reflected over the vertical wedge and the other secondary MR appears. Accordingly, the two secondary MRs interact with each other to form the 3D Mach stem followed by the secondary reflected shock wave surface in the wedge corner. Hence, in this condition a 3D four-shock configuration forms, consisting of the incident shock wave, the 3D Mach stem, the primary reflected shock wave and the secondary reflected shock wave. Meanwhile, a 3D slip surface is observed, which is similar to the slip line structure in a 2D MR. The 3D Mach stem leans forward indicating that it has a greater shock Mach number or higher shock intensity than both of the incident shockwave and the 2D Mach stems. Thus, for the flowfield enclosed in the secondary reflected shockwave surface, density, pressure and temperature are all higher than the ones in the flowfield outside, which are identical to the ones in a 2D reflection case. Therefore, the complex 3D wave configuration appears inside the secondary reflected shock wave surface while the shock wave reflection still obeys the 2D theory in the rest flowfield.

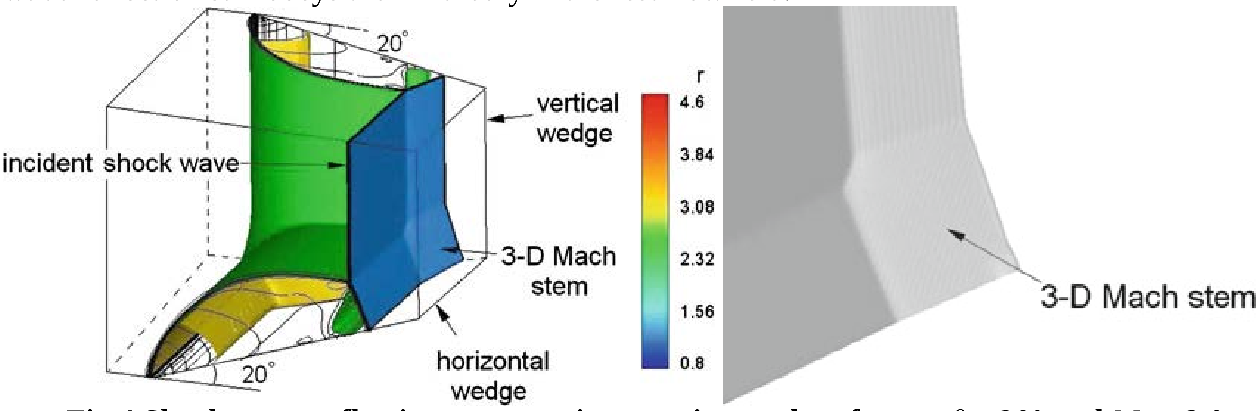


Fig.4 Shock wave reflection over two intersecting wedges for $\alpha = \beta = 20°$ **and** Ms = 3.0

4.2 MR–RR interaction

Fig. 5 shows the interaction configuration over two intersecting wedges with $\alpha = 52°$, $\beta = 45°$ and Ms = 3.0. A regular reflection appears on the horizontal wedge while a double-Mach reflection appears on the vertical wedge. According to the 2D analytical method aforementioned, the Mach stem on the vertical wedge is assumed to be two dimensionally reflected over the horizontal wedge with the shock Mach number $M_{\mathrm{m'}}$ = 4.51 and inclination angle θ_{m} = 42.15°, as shown in Fig. 2. Coinciding with the 2D theory, such an assumed 2D reflection is a Mach reflection. The Mach stem A interacts with B resulting in a 3D Mach stem. However, the Mach stem A is much shorter as compared to the 2D reflection case under the same condition. This is because the horizontal wedge has a transverse inclination with regard to the Mach stem on the vertical wedge and thus it is not a complete 2D wedge in the secondary reflection. Actually, the velocity vector in the flowfield downstream the secondary reflected shock wave has a transverse component, which indicates this assumed 2D reflection has an obvious 3D feature.

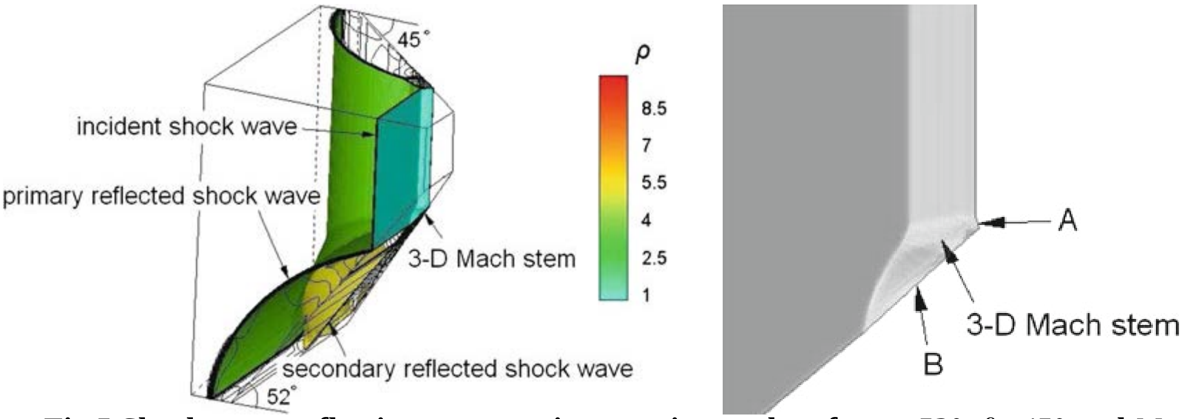


Fig.5 Shock wave reflection over two intersecting wedges for $\alpha = 52°$, $\beta = 45°$ **and** Ms = 3.0

4.3 RR–RR interaction

Fig. 6 shows the shock wave reflection over two intersecting wedges with the wedge angles $\alpha = \beta = 55°$ and incident shock wave Mach number Ms = 3.0. Two regular reflections appear respectively over the two wedges. Obviously, there is not any 3D protuberance structure in the corner of the two intersecting wedges. Here, the primary and secondary reflected shock waves and the incident shock wave originate from the same reflection point which always locates on the wedge intersecting line. Therefore, a three-shock configuration is formed.

4.4 The region of 3D Mach reflection

In summary, analogous to 2D shock wave reflections, 3D shock wave reflections can be generally classified into two categories, i.e., RR and MR. Using 2D shock wave reflection theory, Meguro et al[12] derived the region where the 3D Mach stem exists. However, since the secondary reflection of the Mach stem in the 3D interaction zone is not completely two-dimensional, there are limitations in their derivation.

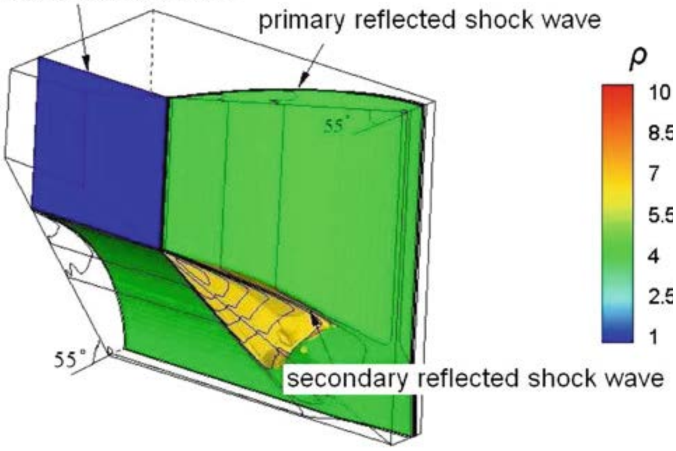


Fig. 6 Shock wave reflection over two intersecting wedges for α = β = 55° and Ms = 3.0

A different wave structure of 3D protuberance forms when an MR–RR interaction occurs in the corner, namely the second type of 3D Mach stem. Fig.7 shows the modified distribution of the 3D shock wave reflection pattern for Ms = 1.5 and Ms = 3.0. The ordinate and abscissa are the inclination angles of the horizontal and vertical wedges, respectively. The dashed lines, which are derived from the detachment criterion, divide the diagrams into three regions: MR-MR interaction, MR–RR interaction and RR–RR interaction. The solid lines denote the boundaries between the solution domains without 3D Mach stem and with the second type of 3D Mach stem. Meanwhile, the dashed–dotted lines separate the solutions of the first and second type of 3D Mach stem. The plots denote the numerical results in the present study. Note that with the increase of the vertical wedge angle β, the dashed–dotted lines bend upward. It is mainly because that for the same α the inclination wedge angle of the assumed 2D reflection, $\angle_m$ [see Fig.1] decreases with the increase of β. Therefore, the reflection of the Mach stem over the horizontal wedge is more likely a MR so that the first type of 3D Mach stem appears.

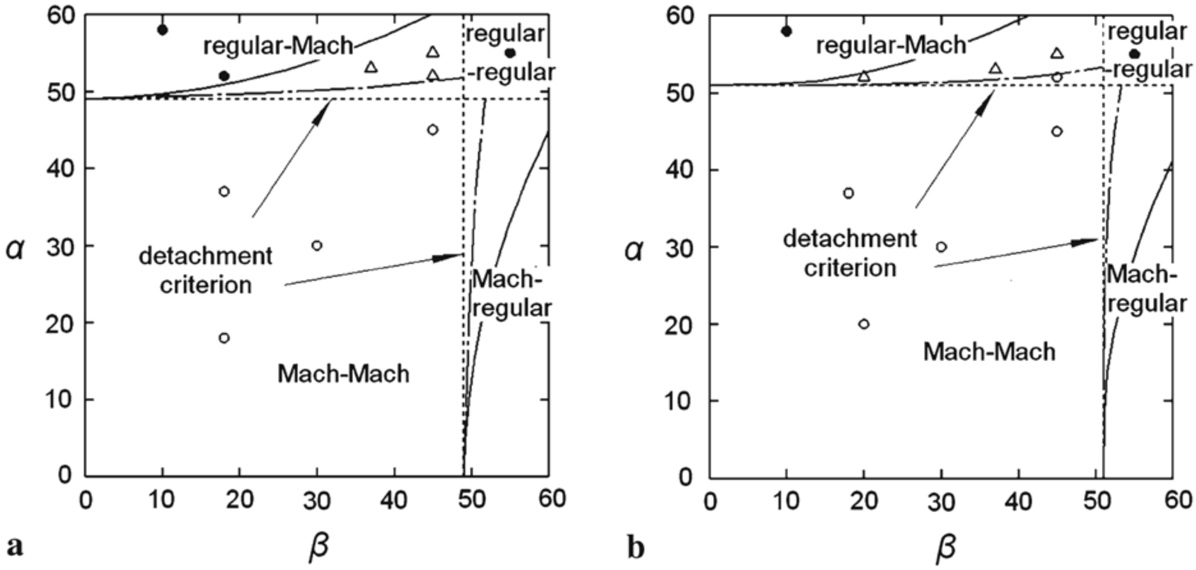
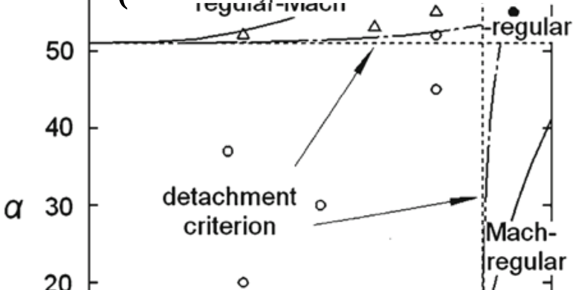


Fig.7 The region of three-dimensional Mach stem: a) M_s = 1.5**; and b)** M_s = 3.0

(○ with first type of 3-D Mach stem △ with second type of 3-D Mach stem ● without 3-D Mach stem **)**



5. Conclusions

The numerical results can be summarized as following:

1. It is found that there are two different kinds of 3D Mach stem structures for the MR–RR interaction, namely the first and the second types of 3D Mach stem, respectively.
2. The 3D Mach stem is a twist surface. If both wedge angles are relatively small, the 3D influence is relatively weak, and the 3D Mach stem twists slightly as a result. On the other hand, if the wedge angles are relatively great, the 3D influence is obvious to twist the 3D Mach stem much severely.
3. A 3D three-shock or four-shock configuration may occur in the intersecting corner. For the MR–MR interaction, the latter forms consisting of the incident shock wave, the 3D Mach stem, the primary and secondary reflected shock waves. In the RR–RR interaction, the incident shock wave, the primary and secondary reflected shock waves meet at the same reflection point to combine a three-shock configuration. For the MR–RR interaction, either of the configurations mentioned above may appear.

Acknowledgments

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References

[1] Mach, E.:Ü ber denVerlauf von Funkenwellen in der Ebene undim Raume. Sitzungsbr Akad Wien 78, 819–838 (1878)

[2] von Neumann, J.: Oblique reflection of shocks, Explos Res Rept 12. Navy Dept Bureau of ordinance, Washington, DC (1943)

[3] von Neumann, J.: Refraction, interaction and reflection of shock waves, NAVORD Rep 203-45. Navy Dept Bureau of ordinance,

[4] Hornung, H.G., Oertel, H., Sandeman, R.J.: Transition to Mach reflexion of shock waves in steady and pseudosteady flow with and without relaxation. J. Fluid Mech. 90, 541–560 (1979)

[5] Chpoun, A., Passerel, D., Li, H., Ben-Dor, G.: Reconsideration of oblique shock wave reflections in steady flows, Part 1. Experimental investigation. J. Fluid Mech. 301, 19–35 (1995)

[6] Ivanov, M., Gimelshein, S., Beylich, A.: Hysteresis effect in stationary reflection of shock waves. Phys. Fluids 7, 685–687 (1995)

[7] Hu, Z.M., Myong, R.S., Kim, M.S., Cho, T.H.: Downstream flow condition effects on the RR MR transition of asymmetric shock waves in steady flows. J. Fluid Mech. 620, 43–62 (2009)

[8] Takayama, K.: Application of holographic interferometry to shock wave research. Proc. SPIE 398, 174–180 (1983) Washington, DC (1943)

[9] Meguro, T., Takayama, K., Onodera, K.: Three-dimensional shockwave reflection over a corner of two intersecting wedges. Shock Wave 7, 107–121 (1997)

[10] Li, H.H.:Experimental and numerical study on unsteady complex flow and wave interaction. PhD Thesis, University of Science and Technology of China, Hefei (2005)