# StabilityProblem of Stadium Roof

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## Abstract

The contribution deals with stability of existing stadium roof. The roof has elliptical shape in the plan view. The height of the roof is variable and is optimally designed according to minimum potential energy. The objective of the paper is the ultimate bearing capacity of the roof loaded with combination of dead loads, snow loads and wind loads. The investigation of the problem was carried out as a static analysis with large deformations. Only the amount of wind load was increased during the analysis. As an iteration procedure capable to handle bending and stability phenomena, the arc-length method was chosen. The solution procedure was stopped by snap-through.

**Keywords:** Stability, Snap-through, Arc-length method, Large deformation.

## Introduction

Big projects in civil engineering give opportunity for development of the theory and application methods. Study of stadium roof bearing capacity, additionally to solution of responsible tasks, brought new experiences, new knowledge in the field of non-linear numerical analysis. From the comprehensive static and dynamic analysis only part concerning stability of the structure is being published in this paper. For that reason, the numerical analysis is presented in application to the structure of the roof. Therefore, the structuring of this paper starts with construction description and continues with stability analysis illustrated with results.

## Structure

The contribution deals with stability of existing stadium roof. The bearing structure of the stadium roof is constructed from the steel beams connected with steel-concrete composite joins.

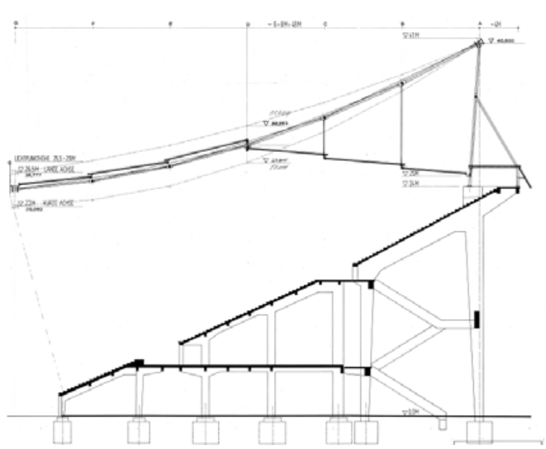

**Figure 1. Sectional view of the structure**

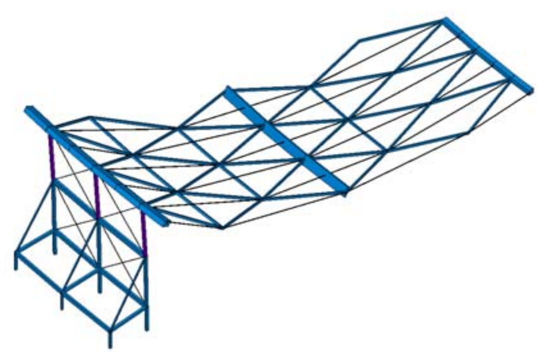

**Figure 2. Section of stadium roof FE model**

The roof has elliptical shape in the plan view. The height of the roof is variable and it is optimally designed according to minimum potential energy.

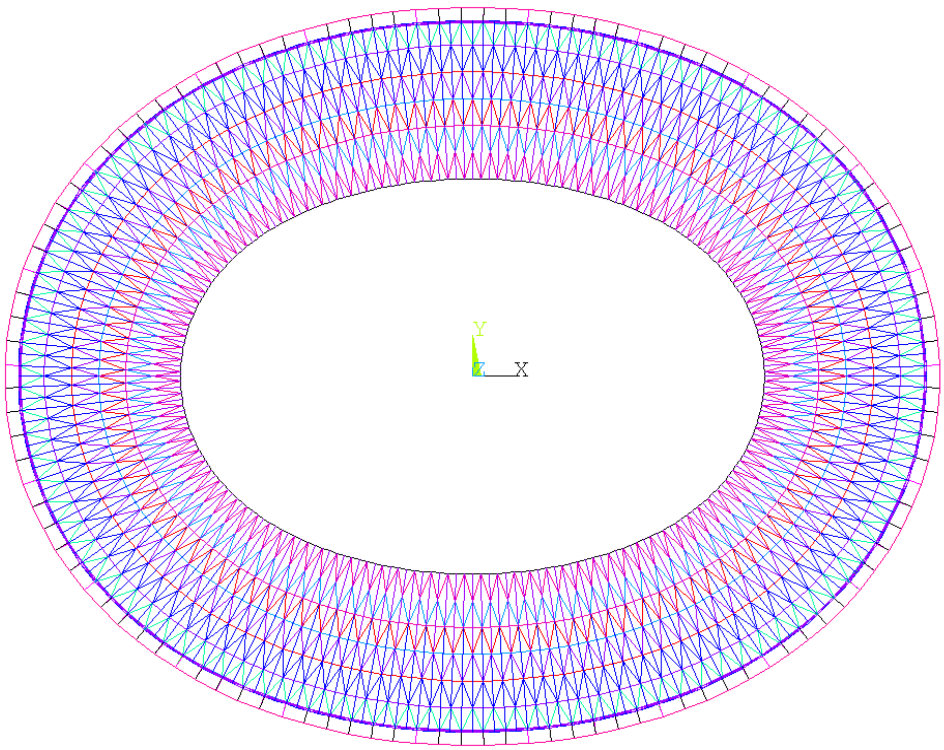


**Figure 3. Plan view of the stadium roof**

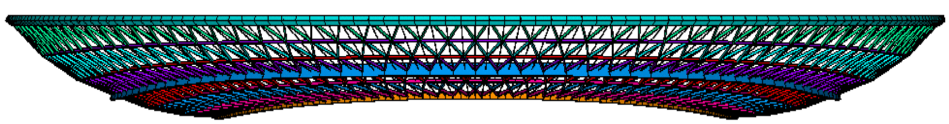

**Figure 4. Side view of the stadium roof**

## Analysis

The main task discussed in this paper is the proof of the ultimate bearing capacity of the stadium roof loaded with combination of dead loads, snow loads and wind loads on the base of Eurocode 1.

### *Eigenvalue Analysis*

The first estimation of the critical loading was solved on the base of eigenvalue problem. The bifurcation point of linear (eigenvalue) buckling curve lies above the limit load obtained from nonlinear snap-through. First natural modes shown in Fig. 5 are useful for the comparison with the results of the nonlinear analysis.

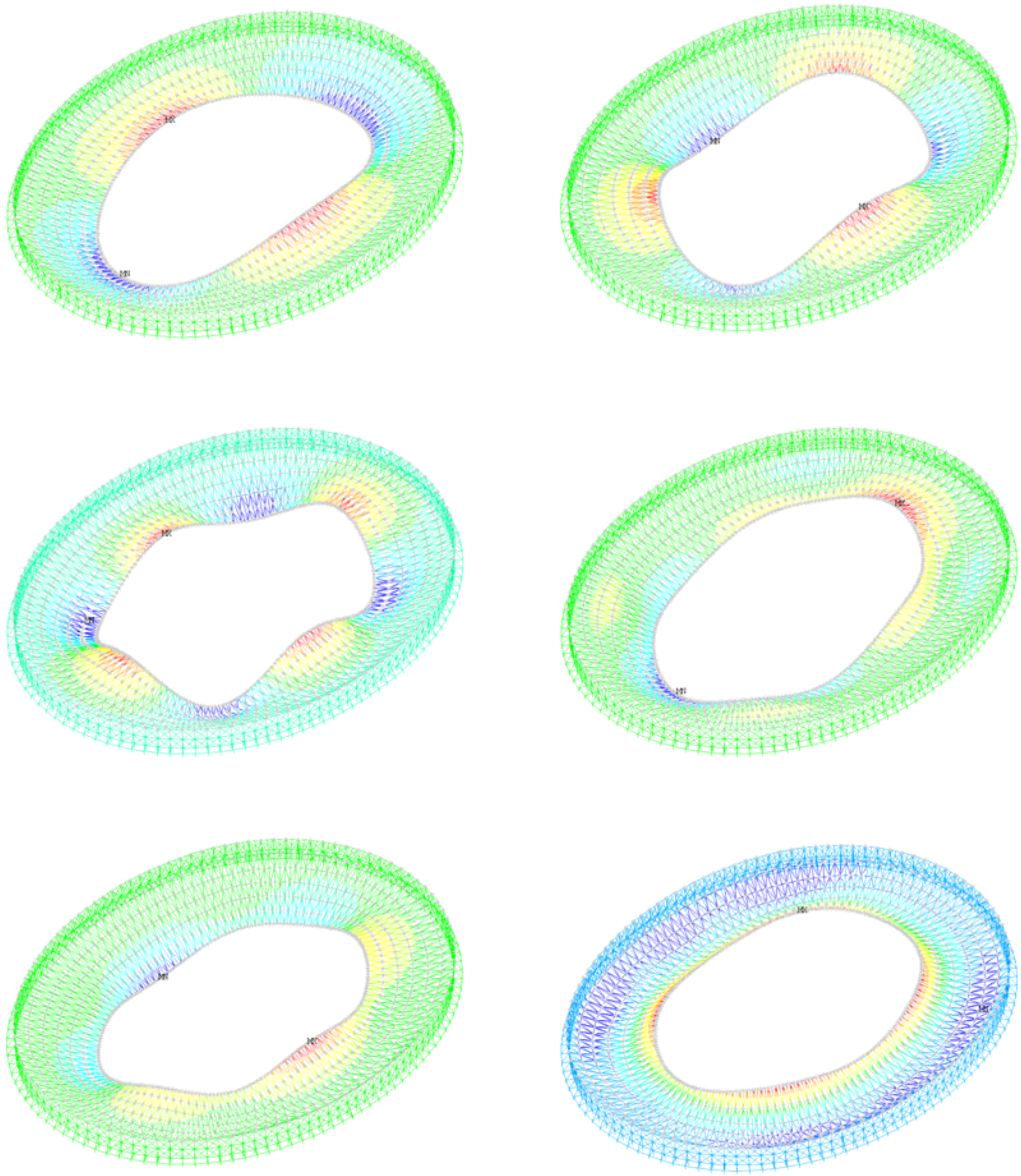

**Figure 5. Shapes of 6 first natural modes**

*Nolinear Buckling Analysis*

The investigation of the snap-through problem was carried out.

*Arc-length Method*

The arc-length method for structural analysis was originally developed by Riks (1972, 1979) and later modified by several authors. In arc-length method in comparison to traditional Newton-Raphson method the constraint curve has the arc geometry. That enables to follow buckling curves in force-displacement diagrams consisting of parts with increasing and decreasing displacement. Using the arc-length method, the equilibrium iterations convergence along an arc and thereby preventing divergence, even when the slope of the load vs. deflection curve becomes zero or negative. The quasi-Newton BFGS method (Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970)) uses approximate Jacobian instead of exact Jacobian for each iteration. Chang (1991) proposed periodically restarted quasi-Newton updates in constant arc-length method. Hellweg and Crisfield (1998) introduced the arc-length method for handling sharp snap-backs. Our analysis used the arc-length method with listed modifications.

*Application of the Arc-length Method*

The load was applied gradually. At each substep the equilibrium iterations were performed to obtain a converged solution. The arc-length method was activated by sets of the minimum and maximum multipliers for the reference arc-length radius. The reference arc-length radius was calculated from the load increment of the first iteration. This increment was determined by total load divided by number of substeps. Lower and upper limit was calculated with help of factors multiplying the difference between reference arc and arc length radius. In each subsequent substep, a new arc-length radius was first calculated based on the arc-length radius of the previous substep and the solution behavior. Next, the newly calculated arc-length radius was further modified so that it falls between the range of the upper limit and lower limit. When the solution did not converge even when using the lower limit of the arc-length radius, the solution terminated.

*Loading of the Structure*

The reference model for the snap-through analysis the roof structure in static equilibrium was chosen. The loading was applied on the base of Eurocode 1: Actions on structures. Two critical load combinations were chosen: Combination 1: dead loads and wind loads and Combination 2: dead loads, snow loads and wind loads. The wind loads were stepwise added.

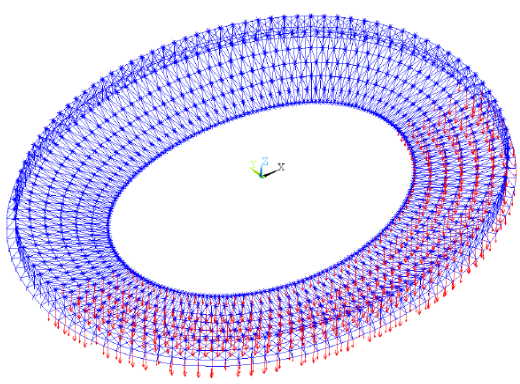

**Figure 5. Snow loading**

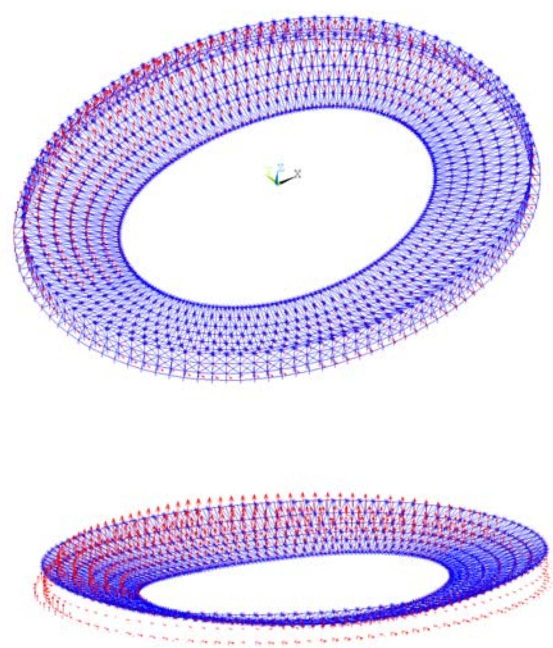

**Figure 6. Wind loads**

## Results

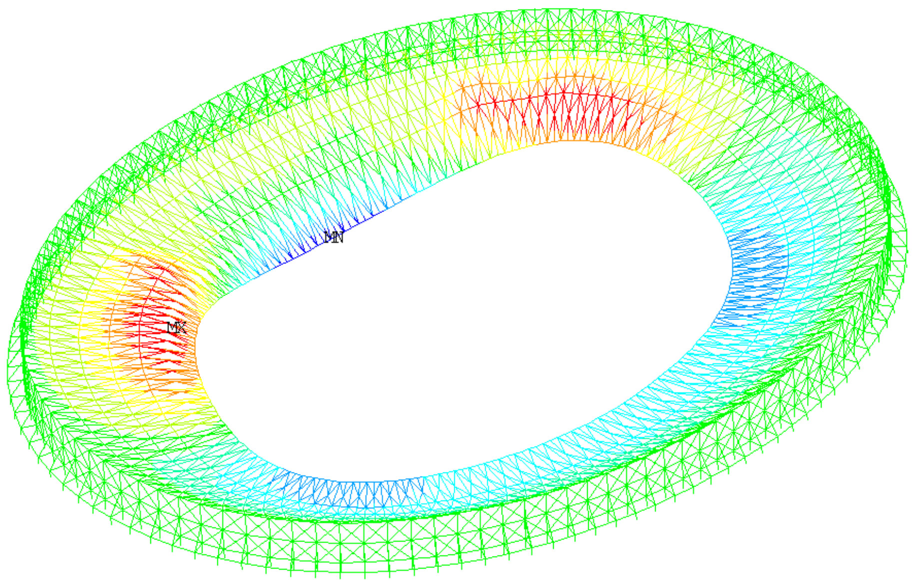


**Figure 7. Shape of snap-through: combination 1 of dead loads and wind loads**

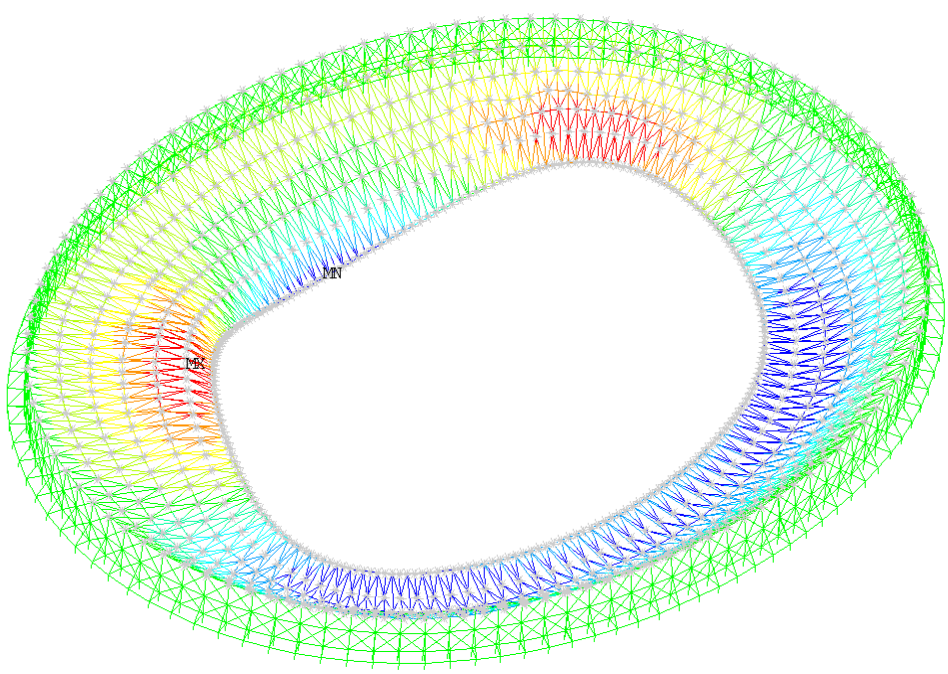


**Figure 8. Shape of snap-through: combination 2 of dead loads, snow loads and wind loads**

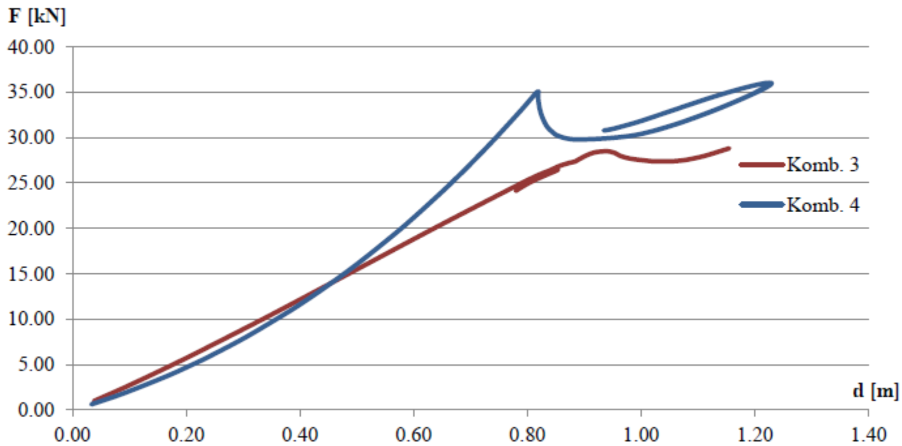


**Figure 9. Force-displacement curve: load case 3 and load case 4 in most displaced nodes**

**Table 1. Identified results**

| Combination Value | Multiplicator for Wind | Max Vertical Displacement [m] |
|---|---|---|
| 1 | 2,3 | 0.85 |
| 2 | 3,0 | 0.82 |

## Conclusions

Successful application of the arc-length method for the proof of the ultimate bearing capacity of real stadium roof loaded with combination of dead loads, snow loads and wind loads was presented. The arc-length method was adjusted for the stiffness of the structure defined on the base of the geometry and structural elements to achieve solution effectively. The ultimate bearing capacity was achieved by snap-through.

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