A rescaling method for generating inflow conditions in simulations of supersonic boundary layers

***Ning Hu[1] and Han-Dong Ma[1]**

[1]China Academy of Aerospace Aerodynamics, 100074 Mailbox 7201-56, Beijing, China.

*Corresponding author: h_ning@pku.org.cn

Abstract

A method for generating turbulent inflow data for simulations of spatially developing boundary layers has been presented. The approach is based on solving for the turbulent mean velocity/temporature profile at the inlet station and mapping the fluctuations from a reference station to the inlet. The mean velocity profile is solved from the Favre-averaged mean momentum equation with the Reynolds stress calculated from a turbulent model proposed by Zhang *et al*. (2012). The mean temperature profile is obtained by applying a generalized Walz's law. LES of adiabatic zero pressure gradient flat plate boundary layer flows at Mach = 2.25 is carried out using fully spatial method with transition region from laminar to turbulent, and also using the inflow condition proposed herein. The boundary layer development and turbulent statistics obtained with the proposed method agree well with the fully spatial approach, with negligible transient section length.

Keywords: boundary layer, supersonic, turbulence, inflow condition

Introduction

The simulation of turbulent boundary layers requires quite detailed inflow information since the resolved flow is unsteady and three-dimensional. Rather than simulating laminar and transitional regions arising near a leading edge, it is often more computationally efficient to formulate a fully turbulent inflow condition. To date, three types of methods for creating appropriate inflow conditions have been suggested: the random fluctuation method (Rai and Moin, 1993), the matching database method (Schlűter *et al*. 2003), and the recycling and rescaling method (Spalart, 1988; Lund *et al*., 1998). Among those methods, the recycling method appears to establish a turbulent shear flow with a fairly short inlet buffer zone and provides accurate downstream profiles.

If the rescaling starts by using downstream data that are far from a correct turbulent state, the skin friction may decrease with time and make the achievement of the desired inflow turbulent state very difficult. To overcome the problem arising from unsuitable initial conditions, Lund *et al*. (1998) suggested making a correction to the resolved velocities during the early part of simulation. Spille-Kohoff and Kaltenbach (2001) suggested adding a source term to the resolved equation based on the desired Reynolds stress. The present paper proposes a new method for recycling and rescaling. In the present method, the position for mapping the reference turbulent field is determined from the value of the order function, instead of using the similarity laws. Without assumption of simple geometrical similarity, the method is easy to be extended to more general flows, with external effects such as pressure gradient or geometrical change, etc.

Rescaling methods

Rescaling order function

The rescaling method is based on the similarity of turbulent boundary layers. The turbulent field at a downstream position can be used as the inlet condition since they have similar turbulent fluctuation ensembles. Usually the rescaling is categorized into the inner scaling

$$U / u_\tau = y^+ \tag{1}$$

and the outer scaling:

$$(U_\infty - U)/u_\tau = f_{\text{wake}}(\eta), \qquad \eta = y/\theta \tag{2}$$

In the presence of pressure gradient, wall heating/cooling or other external forces, the two-layer description no longer holds, and it is difficult to find the correct corresponding position having the similar statistical properties. In this case more universal arguments are needed. According to the SED theory, the statistical properties of a turbulent ensemble can be described by the value of order functions. With a choice of proper order function, the corresponding vertical coordinates between the downstream reference position and the inlet position can still be found. Particularly, in the compressible turbulent boundary layers an order function, which is called Mach-invariant mixing length, can be defined (Zhang *et al*., 2012):

$$\ell^+_{M,MI} = \sqrt{\bar{\rho}^+}\mu^+\ell^+_m = \sqrt{\bar{\rho}^+}\mu^+\frac{(\widetilde{-u''v''}^+)^{1/2}}{\partial\tilde{u}^+/\partial y^+} \tag{3}$$

This function has the length dimension, and describes the turbulent intensity of a vertical position. It is directly defined from Prandtl's mixing length theory but with a profile invariant with Mach number. This Mach-invariance is an important basis for van Driest transformation. According to the SED theory, an order function has multi-layer structure with scaling laws between each two layers, and can be formulated by the multiple of so-called SED base functions:

$$\left(1+(\frac{y^+}{\delta})^p\right)^{n/p} \tag{4}$$

For $\ell^+_{M,MI}$, Zhang *et al*. (2012) give the functional form as

$$\ell^+_{M,MI} = 50(1-(1-(\frac{y}{\delta_{vw}}))^5)(1+(\frac{y^+}{y_1})^{-4})^{-0.5/-4}(1+(\frac{y^+}{y_2})^{-4})^{1/-4} \tag{5}$$

where δ_{vw} is a boundary layer thickness defined as the vertical position where $\langle v'^2\rangle = \langle w'^2\rangle$, $y_1 = 12$ and $y_2 = 85$ are the thicknesses of the sublayer and buffer layer, respectively. Zhang *et al*. (2012) point out that this definition of BL thickness best eliminates out the Mach number dependence.

Velocity rescaling

Denote by subscript *in* the inflow condition and *ref* the condition at the reference *x*- position, the rescaling should guarantee same mean streamwise velocity at *y*- position with the same $\ell^+_{MI,f}$:

$$\frac{U_{in}}{U_{\tau,in}}(\ell^+_{MI,in})U^\alpha_{\tau,in} = \frac{U_{ref}}{U_{\tau,ref}}(\ell^+_{MI,ref})U^\alpha_{\tau,in} \tag{6}$$

where $\ell^+_{MI,ref}$ is calculated from statistics through the spanwise direction, and $\ell^+_{MI,in}$ is given by the theoretical profile (5). The above formulation calls for a knowledge to the wall friction velocity and boundary layer thickness δ_{vw} at the inlet position. When a RANS simulation is used as the inlet boundary, the wall friction velocity is easy to obtain. Here for convenience we just use the empirical law of White *et al*. (1974) instead. Further, since there is no turbulent fluctuation information from the upstream of inlet position, we simply take

$$\delta_{vw} \approx \delta_{99} \tag{7}$$

which is reasonably accurate according to DNS data.

Temperature rescaling

When the mean velocity is properly rescaled, relationship between velocity and temperature rescaling can be used to rescale the temperature. For the mean temperature in a zero-pressure-gradient boundary layer, this relationship can be given by the Walz's equation:

$$\frac{T}{T_e} = \frac{T_w}{T_e} + \frac{T_r - T_w}{T_e}\left(\frac{U}{U_e}\right) - r\frac{\gamma - 1}{2}M_e^2\left(\frac{U}{U_e}\right)^2 \tag{8}$$

where T_r is the recovery temperature, subscript e indicates a freestream quantity, M_e is the freestream Mach number, γ is the ratio of specific heats, and r is the recovery factor. To determine the wall temperature T_w we assume that the ratio between T_w and T_e varies with x in a linear way, which is well supported by the DNS data. Hence, the temperature ratio T_w/T_e can be obtained through the following relation:

$$\frac{T_{w,in}}{T_e x_{in}} - \left(\frac{T_w}{T_e}\right)_{\text{laminar}} = \frac{T_{w,ref}}{T_e x_{ref}} - \left(\frac{T_w}{T_e}\right)_{\text{laminar}} \tag{9}$$

This relationship decouples the mean streamwise velocity and the mean temperature. When fluctuations are small, to a first-order approximation, the mean temperature T and the mean density $\overline{r}$ are related by the state equation $T = p/R\overline{r}$ for perfect gas, where R is the gas constant. Thus, the rescaling of $\overline{r}$ follows that of the mean temperature T is known.

Implementation

The calculation of the mixing length profile requires mean quantities, wherefore a time average is needed to exclude the starting transient if the flow is initialized with a crude guess. In that case, the following formula is used:

$$U^{(m+1)} = w_1 U^{(m)} + w_2 \left\langle U^{(m+1)} \right\rangle_z \tag{10}$$

where $U^{(m+1)}$ and $U^{(m)}$ are the time-averaged mean at time step m + 1 and m, respectively, $\langle u^{(m+1)}\rangle_z$ is the average of u in the spanwise direction at time step m+1, w_1 and w_2 are two weights satisfying $w_1 > 0$, $w_2 > 0$, $w_1 >> w2$, and $w_1 + w_2 = 1$. Lund *et al.* (1998) let w_1 be $1 - (\Delta t/\tau)$ and w_2 be $\Delta t/\tau$, where Δt is the computational time step and τ the characteristic time scale of the averaging interval. From formula (10), we know

$$U^{(m+1)} = w_1^{m+1} U^{(0)} + w_2 (w_1^m \left\langle u^{(1)} \right\rangle_z + w_1^{m-1} \left\langle u^{(2)} \right\rangle_z + \cdots + \left\langle u^{(m+1)} \right\rangle_z). \tag{11}$$

At the beginning of the simulation, because m is small and $w_1 >> w_2$, $U^{(0)}$ takes a very large fraction of $U^{(m+1)}$, as seen from Eq. (11). Thus, we provide a smooth mean profile from TDNS as $U^{(0)}$ instead of using $\langle u^{(0)}\rangle_z$. We choose w_1 so that when the mean information has propagated from the inlet to the recycling station, m is large enough for $U^{(0)}$ to take almost no effect in $U^{(m+1)}$. After the transient, we increase w_1 to run for N steps in order to stabilize the statistics and then switch to a usual running average, i.e., $w_1 = 1 - [1/(N + m - m_0)]$ and $w_2 = 1 / (N + m - m_0)$, where m_0 is the step at which the running average begins. If $U^{(0)}$ is very crude and w_1 is not well attuned, the temporal starting transient can be very long before the right spatial behavior builds up over the boundary layer. If w_1 is too small, a good mean profile $U^{(m+1)}$ cannot be achieved due to insufficiency of effective samples for averaging, which leads to wrong scaling and thus wrong boundary layer mean behavior. A linear interpolation is used to evaluate the right-hand side of Eq. (6) at the mapped coordinates.

Results and discussion

In order to evaluate this inflow method, a turbulent boundary layer with the conditions same with the DNS of Rai *et al*. (1995) is calculated. The freestream Mach number is 2.25. The Reynolds number based on freestream conditions is 6.35×10^5/in. The adiabatic wall temperature is 580ºR, and the temperature at the freestream is 305ºR.

Both fully spatial simulation and simulation using the proposed inflow condition are carried out. The size of the computational domain for both cases is 0.175 in spanwise, and 3 in wall-normal direction. For the fully spatial case, the computational domain consists of a transitional zone, a focus zone and a buffer zone. The transitional zone covers $4 < x < 7$ in the streamwise direction, where x is the distance from the imagined flat plate leading edge. In this region the flow transitions from laminar inflow to turbulence, with blow/suction disturbance at the wall applied within $4.5 < x < 5$. The flow is considered as fully developed turbulence within the focus zone $7 < x < 9$ where statistics are taken, and is followed by a buffer zone $9 < x < 23$. For the case with proposed inflow condition, there is no transitional zone, i.e., the computational domain begins at $x = 7$, and the other zones are the same with the fully spatial case. The reference plane is selected as $x = 9$, where the flow field is rescaled to form the inflow condition at $x = 7$.

Table 1. Parameters of the numerical simulations.

Case	M_∞	$Lx\times Ly\times Lz$	$Nx\times Ny\times Nz$	$\Delta x^+\times\Delta y^+\times\Delta z^+$
A	2.25	$(3+2+14)\times3\times0.175$in	$(586+1264+70)\times55\times256$	**$14.50\times1.05\times6.56$**
B		$(2+14)\times3\times0.175$in	$(1264+70)\times55\times256$	**$14.50\times1.05\times6.56$**

The streamwise grid spacing Δx in the transition zone is no larger than 6.9×10^{-3} and gradually refined to smoothly link the focus zone, where $\Delta x = 1.58\times10^{-3}$. The buffer zone includes 70 grid points and is progressively coarsened in the streamwise direction. In the wall-normal direction the grid extends up to $L_y = 3.0$, with a minimum spacing $\Delta y_w = 1.056\times10^{-4}$. The grid is equally spaced in the spanwise direction, and the width of the domain is $L_z = 0.175$. In wall units (based on the boundary layer properties taken at $x = 8.8$) the mesh spacings in the well-resolved region in the streamwise, wall normal, and spanwise directions are $\Delta x^+ = 14.50$, $\Delta y_w^+ = 1.05$, and $\Delta z^+ = 6.56$, respectively.

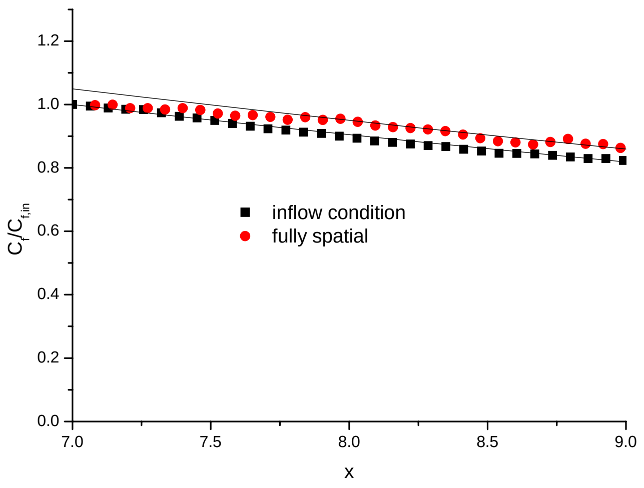
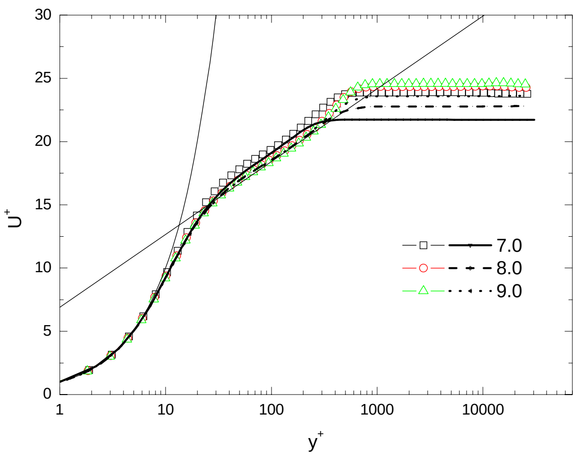


Figure 3: spatial evolution of the friction coefficient $C_f/C_{f,in}$

The rescaling method results in a spatial boundary layer. Figure 3 shows the spatial evolution of the friction coefficient C_f. The rescaling method builds up the spatial boundary layer from the initial periodic flow field after the temporal transient is passed. The skin friction C_f is compared with the result of fully spatial DNS. The development of C_f deviates slightly from the fully spatial DNS. However, the variation of C_f seems to be faster after $x = 8.0$ than before $x = 8.0$ for the fully spatial



DNS, although they are using the same fine grid. This implies that there may be an "early phase" of the turbulent state in the fully spatial DNS where the transition process still has an effect on the flow. After x = 8.0 C_f decays exponentially with the coefficient −0.1. For the simulation using the proposed inflow condition, the exponential decay is valid throughout the whole focus region. Therefore we may consider the development after x = 8.0 in the fully spatial DNS as fully developed, and the proposed method yields fully developed result for the whole field.

Figure 5: van Driest transformed mean streamwise velocity profiles at three stations: lines: DNS with the proposed inflow condition; symbols: fully spatial DNS.

Figure 5 shows the van Driest transformed mean streamwise velocity profiles at three stations. The wall-normal coordinate is also nondimensionalized using wall units. The profiles collapse very well using the transformation and scaling in the logarithmic region and they satisfy the theoretical logarithmic law. Near the inlet boundary, the mean streamwise velocity profiles from both inflow condition and fully spatial DNS deviate slightly from the log-law. However, the deviation of the result from the inflow condition is smaller.

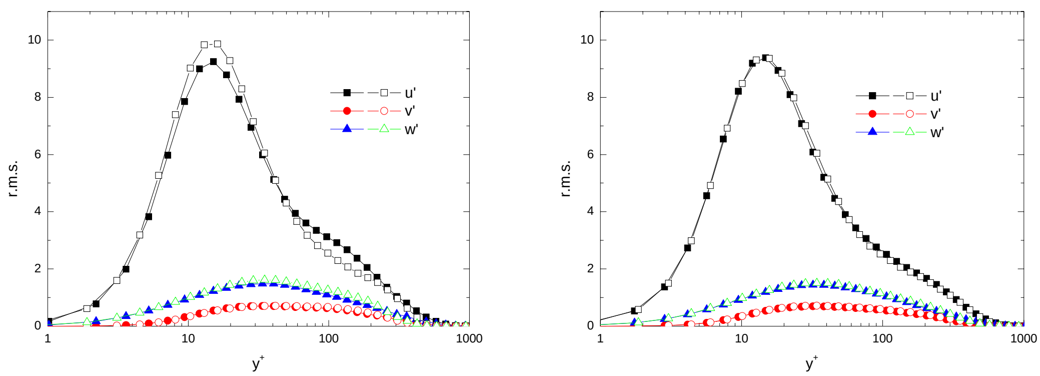


Figure 6: profiles of turbulent intensities compared with DNS. Left: fully spatial; right: proposed inlet condition.

Figure 6 shows the profiles of turbulent intensities at the inlet boundary and at a station within the fully developed region, $x = 8.3$. The profiles in the fully developed region agree well, but at the inlet boundary the two approaches differ. The profiles obtained with the inflow condition, especially the u' profile, are more similar to the fully developed ones.

The highly intermittent boundary layer edge with turbulent bursting events can be appreciated from density distributions in a longitudinal cross-section, Fig. 7. For the inflow condition case and $x > 8$ of the fully spatial case, the virtual boundary thickness does not change much. The structures of the inflow condition case throughout the simulation region have similar shapes. We can say that the proposed method provides good inflow condition for spatial simulation of the turbulent boundary layer.

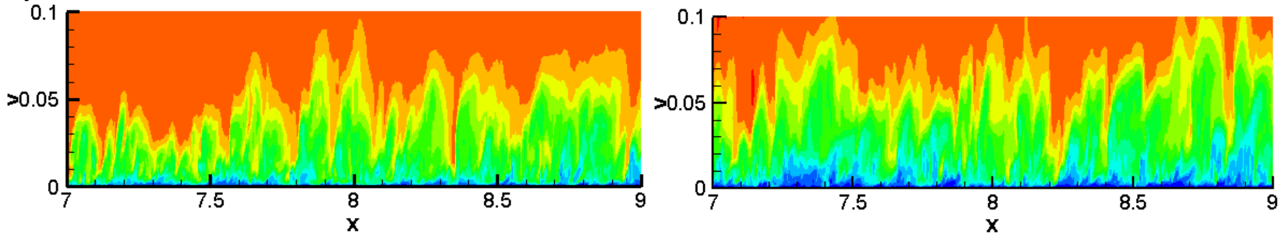


Figure 7: density distributions in a longitudinal cross-section. Left: fully spatial; right: proposed inlet condition.

Conclusions

A method for generating turbulent inflow data for simulations of spatially developing boundary layers has been presented. The approach is based on mapping the time-dependent velocity data from a reference station to the position with the same value of an order function at the inflow station. The selected order function, $\ell^+_{M,MI}$, is Mach number dependent as proposed by Zhang *et al.* (2012). This function has a multi-layer structure with scaling behaviors which describes the structure ensemble properties in each layer. Therefore the position for mapping the reference turbulent field is determined from the value of the order function, instead of using the similarity laws. Direct numerical simulations of a supersonic adiabatic zero pressure gradient flat plate boundary layer flow at Mach = 2.25 are carried out using fully spatial method with transition region from laminar to turbulent, and also using the inflow condition proposed in this paper. The boundary layer development and turbulent statistics obtained with the proposed method agree well with the results of the fully spatial approach, with negligible transient section length. Without assumption of simple geometrical similarity, the method is easy to be extended to more general flows. When subjected to external effects such as pressure gradient or geometrical change, the lengths of scaling regions and the scaling exponents will change, but the turbulent structural ensembles are still characterized by the same order function.

References

Lund T. S., Wu X. and Squires K. D. (1998), Generation of turbulent inflow data for spatially-developing boundary layer simulation. *J. Comput. Phys.* 140, pp. 233.

Rai, M. M. and Moin P. (1993), Direct numerical simulation of transition and turbulence in a spatially evolving turbulence. *J. Comput. Phys*. 109, pp. 169.

Rai M. M., Gatski T. B. and Erlebacher G. (1995) Direct numerical simulation of spatially evolving compressible turbulent boundary layers. *AIAA Paper* 95-0583,.

Schlűter, J. U., Pitsch H. and Moin P. (2003), On boundary conditions for LES in coupled simulations. *AIAA Paper* 2003-0069,.

Spalart P. R. (1988), Direct numerical simulation of turbulent boundary layer up to $Re_\theta = 1410$. *J. Fluid Mech*. 187, pp. 61.

Spille-Kohoff A. and Kaltenbach H. -J. (2001), Generation of turbulent inflow data with a prescribed shear-stress profile, in: 3rd AFOSR Conference, Arlington, TX, pp. 319.

White F. M. (1974), *Viscous Fluid Flow*. McGraw-Hill Book Company.

Zhang, Y.S., Bi, W.T., Hussain, F., Li, X. L., She, Z. S. (2012), A Mach number invariant mean velocity profile of compressible turbulent boundary layers. *Phys. Rev. Lett.* 109, pp. 054502