Coupled Reliability and S-version Finite-Element Model for Probabilistic Distribution of Surface Crack Growth under Constant Amplitude Loading

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Abstract

This research work was focused on probabilistic finite-element analysis of a surface crack growth distribution and its sustainabality. Implementation of S-version Finite Element Model (s-FEM) performs an improvement in the finite-element analysis. The application of s-FEM was by superposed the local dense finite-element mesh on the global coarse finite-element mesh. An adaptive mesh refinement method implemented to provide local mesh refinement without introducing a transition region. The Monte Carlo method embeds with s-FEM for reliability analysis of the structural system with a combination of random parameters. The generated random parameters by Monte-Carlo method activate as an input in the s-FEM sampling process. In order to improve the sampling process, Latin hypercube was implemented and validated with Monte Carlo. Probabilistic research was conducted based on s-FEM results and presented the uncertainty in the model. Numerical example was showed that probabilistic analysis based on s-FEM simulation provides accurate estimation of crack growth distribution. The comparison shows that the association between s-FEM analysis and probabilistic analysis provide an effortless and faithful of quantify the failure probability.

Keywords: Surface crack, S-version Finite Element, Probabilistic, Monte Carlo, Reliability.

Introduction

A crack shape development was first discovered by (Newman Jr and Raju 1981). An assumption was made for an initial semi-elliptical surface crack. It will maintain their shape until a fracture occurs with an increment of crack was based on Paris law (Hou 2011). Numerous studies have investigated the evolution of crack shape through the alternative current field measurement technique, various aspect ratios and the stress intensity factor of corner cracks and round bars. The growth rate of the crack shape has a tendency to slow down at the free surface, and hence the usage of a semi-elliptical crack shape in the simulation process allowed it to evolve in a way that was close to reality. Then the crack shape evolution was independent of the semi-elliptical shape.

Predictions of crack shape and failure of a structure were a challenging problem in fatigue analysis. This was due to the uncertainty in parameter, complicated meshing technique and expensive computation process. Traditionally, the randomness of parameters was considered in safety factor approach and re-meshing process was constructed for the whole domain. It leads to time consuming computing process when the whole structure was involved in a re-meshing process for each crack growth. Therefore, analysis that considers any variation in parameter with feasible meshing technique is needed.

The development of the two-dimensional finite-element analysis has been established for the last few decades (Hou 2004). Nonetheless, modelling intricate geometries is one of the major challenges, due to the high gradient solution in a complex geometry. In order to enhance the finite element solution, adaptive *h*- and *p*-methods are introduced. The *h*-method keeps the order of the elements' approximation and subdivides them into smaller sizes. Meanwhile, the *p*-method increases the order of the approximation while maintaining element size. The efficiency of these methods still needs to be improved by subdividing the finite element mesh using the *h*-method and increasing the degree of the polynomial using the *p*-method. Therefore, (Fish 1992) introduced *hp* procedures, in order to integrate the improvement of polynomial order and mesh refinement. The *hp*-method, the so-called S-version of Finite Element Model (S-FEM) consists of superimposing elements between a local and a global finite element mesh.

The S-FEM has been applied to a diverse range of applications such as heat-affected zone material (Kikuchi et al. 2010; Kikuchi et al. 2012), corrosion cracking (Kikuchi et al. 2011), the crack closure effect (Kikuchi et al. 2010) and composite material (Angioni et al. 2012). Various types of load behaviours (Kikuchi et al. 2010; Kikuchi et al. 2010; Kikuchi et al. 2010; Kikuchi et al. 2011) have become an issue in numerical implementation. The most remarkable is fatigue loading since it represents actual loading cases in practice. Fatigue loading is a leading cause of fracture in structures due to long-term cyclic loading. The integrity of the structure can be questioned when a crack is discovered in a structure. The sustainability of the structure needs to be evaluated in order to avoid a disaster, especially when a crack is detected. Surface cracks are frequently found in aeronautical panels, extrusion press cylinders, riveted aeronautic reinforcements and pressure vessels due to random loading, material, the environment, and so on.

In this paper, probabilistic S-FEM (ProS-FEM) was developed together with fatigue analysis of surface cracking. The main objective of this paper was to evaluate the growth of surface cracks under bending cyclic loads while considering uncertainty in the parameters. A fatigue load was applied throughout the three-dimensional simulation model. The calculation of the stress intensity factor was based on the virtual crack closure technique. The prediction of the crack growth rate was based on Paris's law. In order to predict the range of crack shape development, a new approach is presented in this paper. Experimental works were carried out to validate the simulation data. A comparison between crack growth obtained by the above technique and experimentally is presented and discussed.

Probabilistic s-FEM Finite Element Model

A model as shown in Figure 1 was used in the simulation process. The model was selected based on the actual problem of an aircraft wing (Iyyer et al. 2007). A surface crack was introduced at the centre of the model. Span of loading points was 70 mm length with surface crack at the centre. Two constraints were applied at the bottom end of the model.

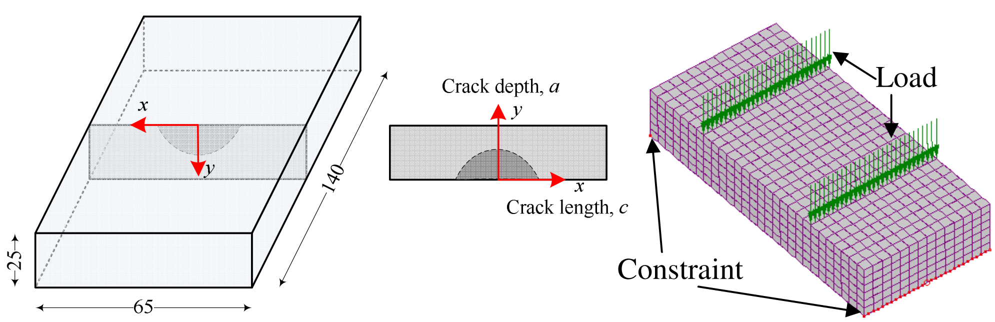

Figure 1. Geometry of the model in mm and axis identification.

Figure 2 shows the concept of S-FEM implementation in crack surface analysis. A coarser mesh was generated for the global mesh while a denser mesh was used in the vicinity of the crack tip area. During the implementation of the global mesh, Ω^G implementation, the crack tip area was neglected temporarily to allow mesh generation for the whole domain. Subsequently, the mesh around the crack tip area was taken into account during the implementation of the local mesh. Then, the local mesh, Ω^L,was overlaid on the global mesh. Finally, the complete structure was ready for analysis. Deciding on the size of the local mesh area was crucial since the propagation of the crack was affected by the calculation of the displacement function. The displacement in the overlaid area was calculated from the global and local meshes as shown below:

$$u(x) = \begin{cases} u^G(x) & x \in \Omega^G - \Omega^L \\ u^G(x) + u^L(x) & x \in \Omega^L \end{cases} \tag{1}$$

On the other hand, the strain of the superimposed area was calculated as below:

$$\varepsilon(x) = \varepsilon^G(x) + \varepsilon^L(x) \tag{2}$$

The final matrix form for S-FEM was:

$$\begin{bmatrix} K_{GG} & K_{GL} \\ K_{LG} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_G \\ u_L \end{Bmatrix} = \begin{Bmatrix} F_G \\ F_L \end{Bmatrix} \tag{3}$$

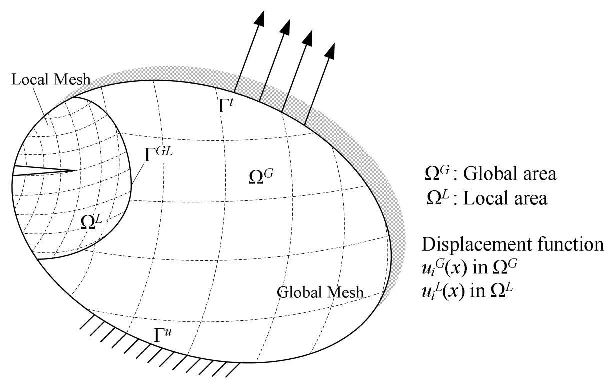

Figure 2. Concept of S-FEM

The $\left[K^{GL}\right]$ matrix represents the stiffness matrix of the superimposed area. By computing the final form of the S-FEM matrix, the displacement can be obtained for each node. The displacement was calculated simultaneously for the global and local meshes for each node. The global mesh was not affected by the changing of the local mesh size. A re-meshing process can be generated for the local area alone since the region of interest is in the area of the crack tip. During the crack growth simulation, the local mesh's size was expanded and the stress intensity factor (SIF) was calculated.

Since the structure is subject to a fatigue load, the crack growth rate is expressed by Paris's law equation:

$$\frac{da}{dN} = C(\Delta K_{eq})^n \tag{4}$$

where *a* and *N* are crack length and number of cycles, respectively. The *C* and *n* coefficients are material constants. The value of ΔK_{eq} is the parameter associated with the fatigue crack growth rate under mixed-mode conditions. Numerous parameters have been proposed for this purpose, but the equivalent stress intensity factor, ΔK_{eq} was used in this research work. The equivalent stress intensity factor, ΔK_{eq} based on Richard's criterion is expressed by:

$$\Delta K_{eq} = \frac{\Delta K_I}{2} + \frac{1}{2}\sqrt{\Delta K_I^2 + 4\left(1.155\Delta K_{II}\right)^2 + 4\left(\Delta K_{III}\right)^2} \tag{5}$$

Furthermore, the crack growth angle was calculated according to the criterion proposed by (Richard et al. 2005):

$$\varphi_o = \mp\left[140^o \frac{\mid K_{II}\mid}{K_I + \mid K_{II}\mid + \mid K_{III}\mid} - 70^o\left(\frac{\mid K_{II}\mid}{K_I + \mid K_{II}\mid + \mid K_{III}\mid}\right)^2\right] \tag{6}$$

where $\varphi_o < 0^o$ for $K_{II} > 0$ and $\varphi_o > 0^o$ for $K_{II} < 0$ and $K_I \geq 0$.

The probabilistic analysis was performed using the Monte-Carlo method (Beer and Liebscher 2008). The material parameters and the initial crack size were deemed to be the random variables. The parameters' distribution was varied for each variable as shown in Table 1. The fatigue analysis was performed in ProS-FEM utilizing the crack closure effect and the appropriate four-point bending model. The embedded probabilistic and fatigue analyses in S-FEM produced a new contribution entitled ProS-FEM.

The parameters for the input-induced closure model for the aluminium alloy 7075-T6 were based on the work by (Liu and Mahadevan 2009). The distributions of the input parameters were developed from available literature data for aluminium alloy 7075-T6. In the probabilistic analysis, the Paris coefficient, *C* of Al 7075-T6 was represented by a mean of 6.54×10^{-13} m/cycle with a standard deviation of 4.01×10^{-11} m/cycle. The distribution was assumed to be lognormal based on the assumptions made in the literature (Liu and Mahadevan 2009). There was no standard deviation for the fatigue power parameter *n* since it was set as deterministic. The reason for this was to control the acceleration of the crack growth in the numerical calculation.

Table 1. Input distribution for the model.

Variable	Distribution	Mean	Standard deviation
Tensile Strength, Ultimate	Deterministic	572 MPa	0
Fatigue power parameter, *n*	Deterministic	3.8863	0
Tensile Strength, Yield	Deterministic	691 MPa	0
Young's modulus, *E*	Normal	71.7 GPa	10.34
Paris coefficient, *C*	Lognormal	6.54×10^{-13} m/cycle	4.01×10^{-13}
Threshold value, ΔK_{th}	Lognormal	5.66 MPa.m$^{0.5}$	0.268
Initial crack length, *da*	Lognormal	0.23 mm	0.05

Once the random variable had been generated, based on its distribution, ProS-FEM started to process the SIF calculation. A random variable of each parameter was generated for one sample. Each sample maintained the same value of the generated variable until the end of crack growth.

Consequently, the calculation of the cycle proceeded with a random maximum crack length da_{max}. Therefore, based on Paris's law, the mean of the cycle can be obtained as

$$E[dN] = \frac{E[da_{max}]}{E[C](\Delta K_{eq_{max}})^n} \tag{7}$$

The equivalent stress $\Delta K_{eq_{max}}$ treated as deterministic since a deterministic load was applied in the analysis. The deterministic load produces constant values of ΔK_I,ΔK_{II} and ΔK_{III} and reflected to the equivalent stress $\Delta K_{eq_{max}}$.

Since da_{max} and C are independent variables, the variance of dN can be calculated as

$$Var[dN] = \left[\frac{1}{(\Delta K_{eq_{max}})^n}\right]^2 \times \left[E[da_{max}]^2 Var[\frac{1}{C}] + E[\frac{1}{C}]^2 Var[da_{max}] + Var[da_{max}]Var[\frac{1}{C}]\right] \tag{8}$$

The random crack length of the crack front was calculated based on the cycle and variance from the random da_{max}. Crack shape development took place after the crack length calculation of each node at the crack front. The process continued with looping of the generation of the random variable for the next sample. The uncertainty quantification analysis can be produced after a number of samples have been simulated. From the results of the quantification of uncertainty, the reliability analysis was conducted.

In order to improve the sampling process, Latin hypercube was develop. The generation of Monte Carlo samples focused around the mean value, and the distribution was scattered within the range of two standard deviations. Meanwhile, the sampling process for the Latin hypercube was scattered within a number of portions. A sample was taken from each portion to generate the input parameter. In this way, all samples covered every part of the distribution. Otherwise, the Monte Carlo technique covers part of the distribution and most of the outliers are neglected. The effect of the sampling technique affected the results, as described in the following section.

Results and Discussion

The simulation results for growth of the surface crack are compared with the experimental work was shown in Figure 3. A notch introduced in the early stage of the experimental process for the initiation of crack growth is shown clearly. After a cyclic load was applied to the specimen, the pre-cracking was generated as shown in the plot. The pre-cracking area was drawn approximately on the graph. Crack growth in the experimental work started to be observed after the pre-cracking took place. The same initial boundary conditions were introduced for the numerical model. The size of the pre-cracking area was modelled in a local mesh. The variations in crack growth as a result of the randomness of uncertain parameters are shown in this figure. It shows all possibilities of crack growth. The trends of crack depth and crack length distribution are presented in this graph. The trends of crack depth and length show the highest frequency in the middle of the distribution. This is due to the effect of randomness in input distribution. As the input distribution was scattered over a range with the highest frequency at the mean, it reflected the crack to growth in the middle of the distribution.

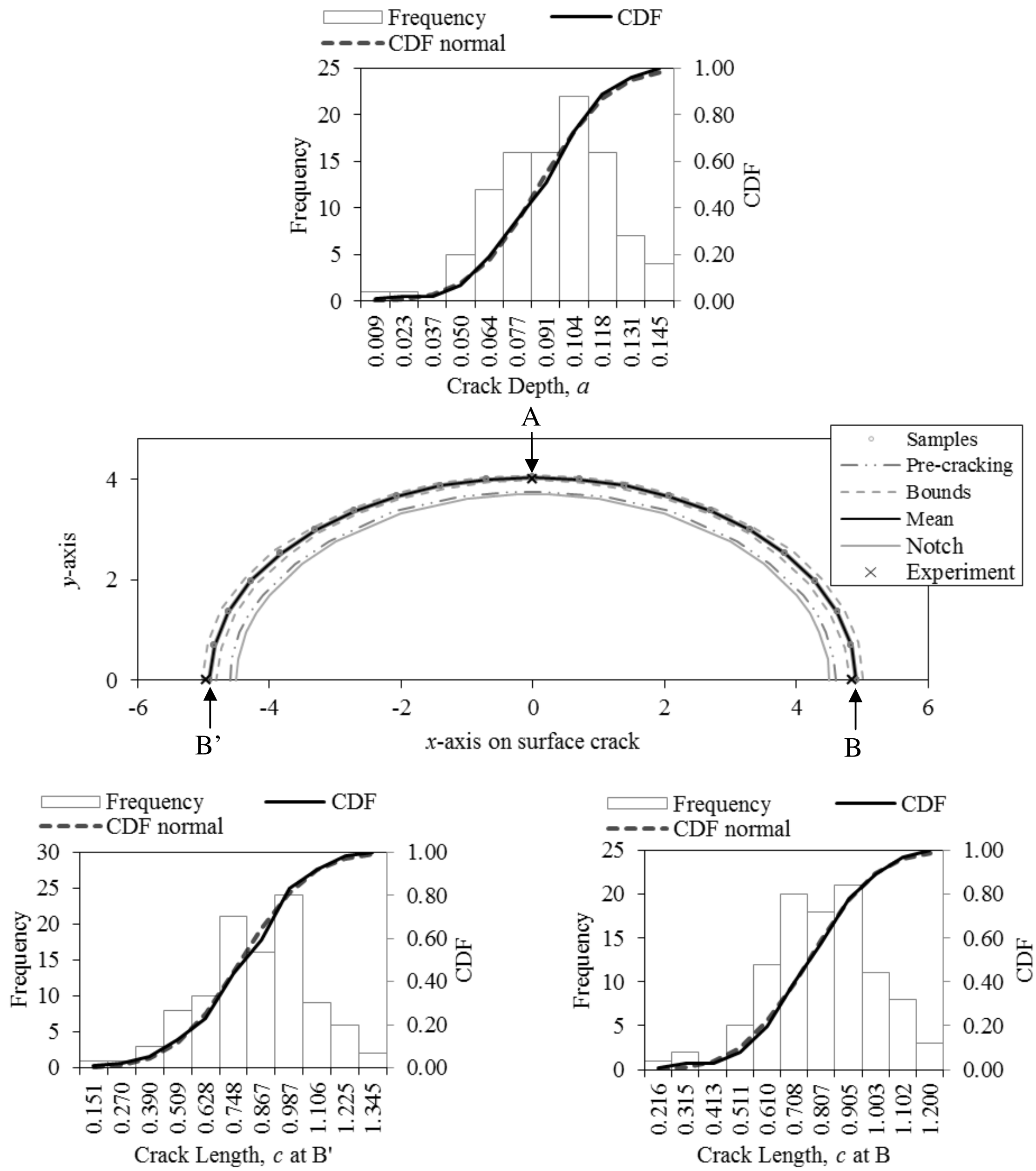

Figure 3. Crack growth on surface crack with length and depth distribution.

Figure 4 shows the reliability result of a four-point bending structure. The failure probability of this structure was 0.18. In general, the number of failed samples increased in proportion to the total number of samples, with a failure rate of 18%. In this case, the load's magnitude plays a major role. The strength of the model could sustain a minimum load but the minimum load fluctuated according to the uncertainty in a parameter. Thus, the failed or safe samples in a model varied probabilistically.

Both the Monte Carlo and the Latin hypercube method need more samples at the beginning of the simulation before they converge to a level of failure probability. After more than 500 samples were generated by the system, the failure probability of the Monte Carlo simulation converged at 0.18, while the Latin hypercube converged after the generation of just 300 samples. As the Latin hypercube could reduce the sample number, the simulation time could be shortened.

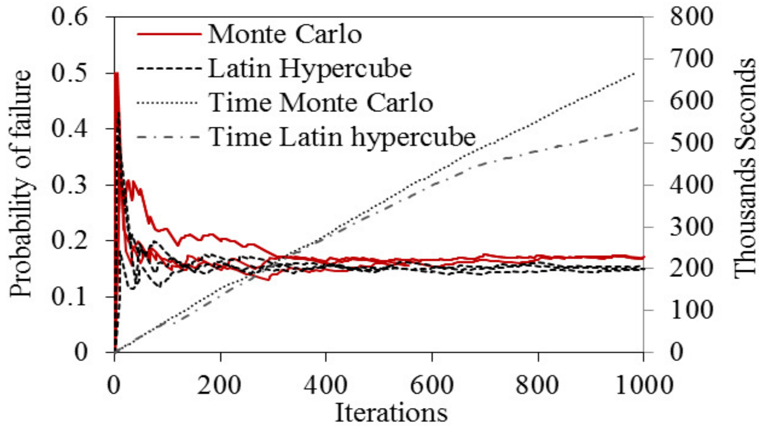

Figure 4. Probability of failure and simulation time for the Monte Carlo and Latin hypercube.

Conclusions

The ProS-FEM simulation technique was developed with auto-mesh generation and a fully automatic fatigue crack growth system. A probabilistic model was developed for four-point bending geometry. The probabilistic scenario of fatigue loading was simulated by treating the Young's modulus, Paris coefficient, threshold value, and initial crack length as variables with distributions. The probabilistic prediction shows good agreement with the experimental results. Furthermore, ProS-FEM provides the distribution of crack length and depth, highlighting the CDF of crack growth. In order to determine the minimum and maximum extent of growth, the mean and bounds of crack growth were generated by ProS-FEM. This shows that the randomness of parameters was modelled successfully in the simulation process. ProS-FEM performed robustly, providing valuable information for analysing high-risk component.

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